# Analysis - Aufgaben: Differentialgleichungen 2

① $t(1+t)\,\dot{x}(t) = x(t)$

a) 1 Ordnung, gewöhnlich, linear,

b) $x(t) = \frac{Ct}{1+t} \Rightarrow \underline{\dot{x}} = \frac{C\cdot(1+t) - Ct\cdot 1}{(1+t)^2}$

$= \frac{C}{(1+t)^2}$

$= \frac{Ct}{t\cdot(1+t)^2} = \underline{\frac{x}{t(1+t)}}$ □

c) $P \in \text{graph}(x) \Leftrightarrow x(1) = 8$

$\Leftrightarrow \frac{C}{2} = 8 \Leftrightarrow \underline{C = 16}$

$\Rightarrow \underline{\underline{x(t) = \frac{16t}{1+t}}}$

② $x(t) = A\cdot\sin(\omega_0 t + \varphi)$

a) $A$ = Amplitude,

$\omega_0$ = Frequenz

$\varphi$ = Phase

b) $\dot{x} = A\cdot\omega_0\cdot\cos(\omega_0 t + \varphi)$

$\ddot{x} = -A\omega_0^2\cdot\sin(\omega_0 t + \varphi)$

$\Rightarrow \ddot{x} + \omega_0^2 x = -A\omega_0^2\sin(\omega_0 t + \varphi) + \omega_0^2\cdot A\sin(\omega_0 t + \varphi)$

$= 0$ □

③ <u>Beh.</u>: $x(t) = 4\cdot e^t(1+e^t)^{-1}$ ist Lsg. des AWP: $(1+e^t)\dot{x} = x$, $x(0) = 2$

<u>Bew.</u>: • $\underline{x(0)} = 4\cdot e^0(1+e^0)^{-1} = \frac{4\cdot 1}{(1+1)} = \underline{2}$ ✓

• $\underline{\dot{x}} = 4\cdot e^t\cdot(1+e^t)^{-1} + 4e^t\cdot(-1)\cdot(1+e^t)^{-2}\cdot e^t$

$= 4e^t(1+e^t)^{-1}\cdot\left(1 - e^t(1+e^t)^{-1}\right)$

$= x\cdot\left(1 - \frac{e^t}{1+e^t}\right) = x\cdot\left(\frac{1+e^t-e^t}{1+e^t}\right) = \underline{x\cdot\frac{1}{1+e^t}}$ □

(3) a) $\dot{x} - 3t = 0 \iff \dot{x} = 3t$

$$\overset{\int \dots dt}{\iff} \quad \underline{\underline{x(t) = \tfrac{3}{2} \cdot t^2 + C}}$$

b) $\ddot{x} = t$

$$\overset{\int \dots dt}{\iff} \quad \dot{x} = \tfrac{1}{2} \cdot t^2 + C_1$$

$$\overset{\int \dots dt}{\iff} \quad \underline{\underline{x(t) = \tfrac{1}{6} \cdot t^3 + C_1 t + C_2}}$$

c) $\dot{x} = \frac{x}{t} \iff \frac{dx}{dt} = \frac{x}{t}$

$$\iff \frac{1}{x}\, dx = \frac{1}{t}\, dt$$

$$\overset{\int \dots}{\iff} \ln|x| = \ln|t| + C_1$$

$$\iff \underline{\underline{x(t)}} = e^{\ln|t| + C_1} = \underline{\underline{C_2 \cdot t}}$$

d) $3t^2 + at - 5\dot{x} = 0 \iff \frac{5\,dx}{dt} = 3t^2 + at$

$$\iff \int 5\,dx = \int 3t^2 + at\; dt$$

$$\iff \underline{\underline{x(t)}} = \frac{t^3 + \tfrac{1}{2} \cdot a t^2 + C_1}{5} = \underline{\underline{\tfrac{1}{5} \cdot t^3 + \tfrac{1}{10} a t^2 + C_2}}$$

e) $\dot{x}(1 + t^2) = tx \iff \frac{dx}{x} = \frac{t}{1+t^2}\, dt$

$$\iff \int \frac{1}{x}\, dx = \int \frac{t}{1+t^2}\, dt$$

subst.: $u(t) = 1 + t^2$

$$\Rightarrow \frac{du}{dt} = 2t \iff dt = \frac{du}{2t}$$

$$\iff \ln|x| + C_1 = \int \frac{1}{2u} \cdot du = \tfrac{1}{2} \cdot \ln|u| + C_2$$

$$\iff \ln|x| = \tfrac{1}{2} \cdot \ln|u| + C_3 \overset{\text{Rücksubst.}}{=} \tfrac{1}{2} \cdot \ln(1 + t^2) + C_3$$

$$\iff \underline{\underline{x(t)}} = e^{\tfrac{1}{2} \cdot \ln(1+t^2) + C_3} = \underline{\underline{C_4 \cdot (1 + t^2)^{1/2}}}$$

f) $\dot{x} = (1-x)^2 \iff \int \frac{1}{(1-x)^2}\,dx = \int 1\,dt$

subst. $u(x) = 1-x$

$\Rightarrow \frac{du}{dx} = -1 \iff dx = -du$

$\iff \int -\frac{1}{u^2}\,du = \int 1\,dt$

$\iff u^{-1} = t + C$

Rück-subst. $\iff \frac{1}{1-x} = t + C$

$\iff x(t) = 1 - \frac{1}{t+C}$

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a) $x\dot{x} + 1 = t \quad , \quad x(-1) = 2$

$\iff x \cdot \frac{dx}{dt} = t - 1$

$\iff \int x\,dx = \int t-1\,dt$

$\iff \frac{1}{2}x^2 = \frac{1}{2}t^2 - t + C_1$

$\iff \underline{x(t) = (t^2 - 2t + C_2)^{1/2}} \quad , \quad x(-1) = (1 + 2 + C_2)^{1/2} = 2$

$\iff \underline{C_2 = 1}$

$\Rightarrow \underline{\underline{x(t)}} = (t^2 - 2t + 1)^{1/2} = ((t-1)^2)^{1/2} = \underline{\underline{|t-1|}}$

b) $\dot{x} + \cos t \cdot x = 0 \quad , \quad x(\pi/2) = 2\pi$

$\iff \frac{dx}{dt} = -\cos t \cdot x$

$\iff \int \frac{1}{x}\,dx = \int -\cos t\,dt$

$\iff \ln|x| = -\sin t + C_1$

$\iff \underline{x(t) = C_2 \cdot e^{-\sin t}} \quad , \quad x(\pi/2) = C_2 \cdot e^{-1} = 2\pi$

$\iff \underline{C_2 = e \cdot 2\pi}$

$\Rightarrow \underline{\underline{x(t)}} = e \cdot 2\pi \cdot e^{-\sin t} = \underline{\underline{2\pi \cdot e^{1-\sin t}}}$

⑥ a) $\dot{x} = 1 + 2\cdot\left(\frac{x}{t}\right)$

<u>Subst.</u> $u = \frac{x}{t} \iff x = u\cdot t$

$\Rightarrow \dot{x} = \dot{u}t + u$

$\Rightarrow \dot{u}t + u = 1 + 2u$

$\iff \frac{du}{dt}\cdot t = 1 + u$

$\iff \int \frac{1}{1+u}\,du = \int \frac{1}{t}\,dt \iff \ln|1+u| = \ln|t| + C_1$

$\Rightarrow |1+u| = e^{\ln|t| + C_1} = C_2\cdot t$

Rücksubst. $\Rightarrow$ $\underline{\underline{x(t) = C_2 t^2 - t}}$

b) $\dot{x} = (t + x + 1)^2$

<u>Subst.</u> $u = t + x + 1 \iff x = u - t + 1$

$\Rightarrow \dot{x} = \dot{u} - 1$

$\Rightarrow \dot{u} - 1 = u^2$

$\iff \int \frac{1}{1+u^2}\,du = \int 1\,dt \iff \arctan u = t + C_1$

$\iff u = \tan(t + C_1)$

Rücksubst. $\Rightarrow$ $\underline{\underline{x(t) = \tan(t + C_1) - t - 1}}$