Lösungen zu **Analysis-Aufgaben:** *Differentialrechnung 4*

① a) 4 b) nicht-def. c) nicht-def. d) nicht-def.

e) -16 f) -1 g) 1 h) nicht-def.

② a) $(f\circ g)(x) = \cos(x^3) \Rightarrow \underline{(f\circ g)'(x)} = -\sin(x^3)\cdot 3x^2 = \underline{\underline{-3x^2\cdot\sin(x^3)}}$

b) $(g\circ f)(x) = \cos^3 x \Rightarrow \underline{(g\circ f)'(x)} = 3(\cos x)^2\cdot(-\sin x) = \underline{\underline{-3\cos^2 x\cdot\sin x}}$

c) $(h\circ f)(x) = e^{\cos x} \Rightarrow \underline{(h\circ f)'(x)} = e^{\cos x}\cdot(-\sin x) = \underline{\underline{-\sin x\cdot e^{\cos x}}}$

d) $(h\circ h)(x) = e^{e^x} \Rightarrow \underline{\underline{(h\circ h)'(x) = e^{e^x}\cdot e^x = e^{e^x+x}}}$

③ a) $\underline{\underline{a'(x)}} = 2\cdot(1-2x)\cdot(-2) = \underline{\underline{-4(1-2x)}}$

b) $\underline{\underline{b'(x)}} = 4\cdot(1+x^3)^3\cdot 3x^2 = \underline{\underline{12x^2(1+x^3)^3}}$

c) $\underline{\underline{c'(x)}} = (-1)\cdot(9-4x)^{-2}\cdot(-4) = \underline{\underline{4\cdot(9-4x)^{-2}}}$

d) $\underline{\underline{d'(x)}} = 2\cdot\left(\frac{a}{a+x^2-x}\right)'\cdot\left[\frac{0\cdot(a+x^2-x)-a\cdot(2x-1)}{(a+x^2-x)^2}\right] = \underline{\underline{\frac{-2a^2(2x-1)}{(a+x^2-x)^3}}}$

e) $\underline{\underline{e'(x)}} = \frac{1}{2}\cdot(x+x^2)^{-1/2}\cdot(1+2x) = \underline{\underline{(\frac{1}{2}+x)\cdot(x+x^2)^{-1/2}}}$

f) $\underline{\underline{f'(x)}} = e^{\cos x}\cdot(-\sin x) = \underline{\underline{-\sin x\cdot e^{\cos x}}}$

g) $g(x) = (e^{\ln x})^x = e^{x\cdot\ln x} \Rightarrow \underline{\underline{g'(x)}} = e^{x\cdot\ln x}\cdot(1\cdot\ln x + x\cdot\frac{1}{x}) = \underline{\underline{(\ln x+1)e^{x\cdot\ln x}}}$

④ a) - c): Setze $f(x) = 2x+3 \Rightarrow f'(x) = 2$

$g(x) = (4x^2-3x)^4 \Rightarrow g'(x) = 4\cdot(4x^2-3x)^3\cdot(8x-3)$

$\Rightarrow a(x) = f(x)\cdot g(x) \Rightarrow \underline{a'(x)} = 2\cdot(4x^2-3x)^4 + (2x+3)\cdot 4\cdot(4x^2-3x)^3\cdot(8x-3)$

$(= f'\cdot g + f\cdot g')$

$= \underline{(4x^2-3x)^3\cdot[2\cdot(4x^2-3x)+4\cdot(2x+3)\cdot(8x-3)]} = \ldots$

$b(x) = \frac{f(x)}{g(x)} \Rightarrow \underline{\underline{b'(x)}} = \frac{2\cdot(4x^2-3x)^4 - (2x+3)\cdot 4\cdot(4x^2-3x)^3\cdot(8x-3)}{((4x^2-3x)^4)^2}$ $\left(= \frac{f'g-fg'}{g^2}\right)$

$= \underline{\frac{2\cdot(4x^2-3x)-4(2x+3)(8x-3)}{(4x^2-3x)^5}} = \ldots$

$c(x) = (b(x))^4 \Rightarrow c'(x) = 4\cdot\left(\frac{2x+3}{(4x^2-3x)^4}\right)^3\cdot\frac{(-8)(28x^2+39x-18)}{(4x^2-3x)^5}$

$(= 4\cdot(b(x))^3\cdot b'(x))$

$= \underline{\underline{\frac{(-8)(2x+3)^3\cdot(28x^2+39x-18)}{(4x^2-3x)^{17}}}}$

(4) d) $d(x) = 2x^2 \cdot \cos^2(2x)^2$

$$= 2x^2 \cdot \cos(4x^2) \cdot \cos(4x^2) \qquad (= f \cdot g \cdot h)$$

$$\Rightarrow d'(x) = 4x \cdot \cos(4x^2) \cdot \cos(4x^2) + 2x^2 \cdot (-\sin(4x^2) \cdot 8x \cdot \cos(4x^2)$$

$$(= f' \cdot g \cdot h + f \cdot g' \cdot h + 2x^2 \cdot \cos(4x^2) \cdot (-\sin(4x^2) \cdot 8x + f \cdot g \cdot h')$$

$$= 4x \cdot \cos(4x^2) \cdot \left[\cos(4x^2) - 8x^2 \cdot \sin(4x^2)\right]$$

e) $e(x) = \sin x \cdot \cos x + 30 \cdot e^{\omega x} - \dfrac{\sqrt[3]{x^4}}{x^{2/5}}$

$$\sin x \cdot \cos x \rightarrow \cos x \cdot \cos x + \sin x \cdot (-\sin x)$$

$$30 \cdot e^{\omega x} \rightarrow 30\omega \cdot e^{\omega x}$$

$$\frac{\sqrt[3]{x^4}}{x^{2/5}} \rightarrow \frac{14}{15} \cdot x^{-1/15}$$

$$\Rightarrow e'(x) = \cos^2 x - \sin^2 x + 30\omega e^{\omega x} - \frac{14}{15} \cdot x^{-1/15}$$