Analysis-Aufgaben: Integralrechnung 1

①

a) $F(x) = \frac{1}{6}\cdot x^6 + C$ b) $F(x) = 2\cdot e^x + C$ c) $F(x) = x^3 + C$

d) $F(x) = \frac{1}{5}\cdot x^5 + C$ e) $G(x) = \frac{1}{8}\cdot x^2 + C$ f) $G(x) = \frac{4}{7}\cdot x^7 + C$

g) $G(x) = x^n + C$ h) $G(x) = \frac{n}{n+1}\, x^{n+1} + C$

i) $F(x) = -\frac{5}{4}\cdot x^4 + e^x + 5\cdot\cos x + C$ j) $F(x) = \frac{2}{3}\cdot x^{3/2} + C$

k) $F(x) = \frac{1}{3}\cdot e^{3x} + x - \frac{1}{2}\cdot x^2 + C$ l) $F(x) = \ln|x| + C$

m) $G(x) = \sin^{-1}(x) + C$ n) $G(x) = 2\cdot\sin^{-1}(x) + C$ o) $G(x) = \frac{1}{2}\cdot\sin^{-1}(x) + C$

②

a) $F(x) = \frac{1}{4}\cdot x^2 + C$;

$P = (-2/4) \in \text{graph}(F) \iff F(-2) = 4 \iff \frac{1}{4}\cdot(-2)^2 + C = 4 \iff \underline{C = 3}$ $\Rightarrow$ $\underline{\underline{F(x) = \frac{1}{4}\cdot x^2 + 3}}$

b) $F(x) = \frac{1}{3}\cdot x^3 - x^2 - x + C$; $C = 1$ $\Rightarrow$ $\underline{\underline{F(x) = \frac{1}{3}\cdot x^3 - x^2 - x + 1}}$

c) $F(x) = \sin x + x + C$; $C = 0$ $\Rightarrow$ $\underline{\underline{F(x) = \sin x + x}}$

d) $F(x) = C$; $C = 2001$ $\Rightarrow$ $\underline{\underline{F(x) = 2001}}$

③

a) $\int 1 - \frac{1}{3}x\,dx = \underline{\underline{x - \frac{1}{6}x^2 + C}}$ b) $\int 5x^4 - 3x^2\,dx = \underline{\underline{x^5 - x^3 + C}}$

c) $\int \frac{1}{t^4} + \frac{1}{2}t^2 + 1\,dt = \int t^{-4} + \frac{1}{2}\cdot t^2 + 1\,dt = \underline{\underline{-\frac{1}{3}t^{-3} + \frac{1}{6}t^3 + t + C}}$

d) $\int \cos t\,dt = \underline{\underline{\sin t + C}}$ e) $\int \frac{1}{x^2+9}\,dx \overset{(11)}{=} \underline{\underline{\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C}}$

f) $\int abc\,dx = \underline{\underline{abcx + C}}$ g) $\int abc\,da = \underline{\underline{\frac{1}{2}\cdot a^2bc + C}}$

h) $\int 2r\sin r^2\,dr = \underline{\underline{-\cos r^2 + C}}$ i) $\int 2\cdot\sin x\cdot\cos x\,dx = \underline{\underline{\sin^2 x + C}}$

j) $\int -0.25\cos x\cdot\sin x\,dx = \underline{\underline{\frac{1}{8}\cdot\cos^2 x + C}}$

k) $\int e^x\cdot\sin 2x\,dx \overset{(15)}{=} \underline{\underline{\frac{1}{5}\cdot e^x\cdot(\sin 2x - 2\cdot\cos 2x) + C}}$

l) $\int 2\cdot e^{3x}\cdot\cos(4x+5)\,dx = \underline{\underline{2\cdot\frac{e^{3x}}{25}\cdot\big(3\cdot\cos(4x+5) + 4\cdot\sin(4x+5)\big) + C}}$