Integralrechnung 2

Joanne Büchi ①

1. a) $\int_1^e x \cdot \ln x \, dx$ setze: $g(x) = \ln x \Rightarrow g'(x) = \frac{1}{x}$
$f'(x) = x \Rightarrow f(x) = \frac{x^2}{2}$

$= \frac{x^2}{2} \cdot \ln x \Big|_1^e - \int_1^e \frac{x^{\cancel{2}}}{2} \cdot \frac{1}{\cancel{x}} \, dx = \frac{e^2}{2} \cdot \ln(e) - \frac{1}{2} \cdot \ln(1) - \underbrace{\left(\frac{x^2}{4}\right)\Big|_1^e}_{\frac{e^2}{4} - \frac{1}{4}}$

$= \frac{e^2}{2} - \frac{e^2}{4} + \frac{1}{4} = \frac{2e^2}{4} - \frac{e^2}{4} + \frac{1}{4} = \frac{e^2}{4} + \frac{1}{4} = \underline{\underline{\frac{1}{4}(e^2+1)}}$

b) $\int_0^1 x \cdot e^{2x} \, dx$ $g(x) = x \Rightarrow g'(x) = 1$
$f'(x) = e^{2x} \Rightarrow f(x) = \frac{e^{2x}}{2}$

$= \frac{e^{2x}}{2} \cdot x \Big|_0^1 - \int_0^1 \frac{e^{2x}}{2} \, dx = \frac{e^2}{2} - \frac{e^{2x}}{4}\Big|_0^1 = \frac{e^2}{2} - \left(\frac{e^2}{4} - \frac{1}{4}\right)$

$= \frac{2e^2}{4} - \frac{e^2}{4} + \frac{1}{4} = \frac{e^2}{4} + \frac{1}{4} = \underline{\underline{\frac{1}{4}(e^2+1)}}$

c) $\int_0^{\pi} x \cdot \cos(x) \, dx$ $g(x) = x \Rightarrow g'(x) = 1$
$f'(x) = \cos(x) \Rightarrow f(x) = \sin(x)$

$= \sin(\pi) \cdot \pi - \sin(0) \cdot 0 - \int_0^{\pi} \sin(x) \, dx = -(-\cos(x))\Big|_0^{\pi} = \cos(\pi) - \cos(0)$
$= \underline{\underline{-2}}$

d) $\int_0^1 (1+x) \cdot e^x \, dx$ $g(x) = 1+x \Rightarrow g'(x) = 1$
$f'(x) = e^x \Rightarrow f(x) = e^x$

$= e^1 \cdot 2 - 1 - \int_0^1 e^x \cdot 1 \, dx = 2e^1 - 1 - (e^1 - 1) = e - 1 + 1 = \underline{\underline{e}}$

e) $\int_1^2 x^{-2}\cdot\ln(x)\,dx$ $\qquad g(x)=\ln(x) \Rightarrow g'(x)=\frac{1}{x}$

$\qquad f'(x)=x^{-2} \Rightarrow \frac{x^{-1}}{-1}=f(x)$

$$= \frac{x^{-1}}{-1}\cdot\ln(x)\Big|_1^2 - \int_1^2 \frac{x^{-1}}{-1}\cdot x^{-1}\,dx = \frac{1}{-2}\cdot\ln(2) - \int_1^2(-x^{-2})\,dx$$

$$= -\frac{1}{2}\cdot\ln(2) - (x^{-1})\Big|_1^2 = -\frac{1}{2}\cdot\ln(2) - \left(\frac{1}{2}-1\right) = -\frac{1}{2}\cdot\ln(2)+\frac{1}{2} = \underline{\underline{\frac{1}{2}(1-\ln(2))}}$$

1

f) $\int_0^{\pi/4}\frac{x}{\cos^2(x)}\,dx$ $\qquad g(x)=x \Rightarrow g'(x)=1$

$\qquad f'(x)=\frac{1}{\cos^2(x)} \rightarrow f(x)=\tan(x)$

$$= \tan(x)\cdot x\Big|_0^{\pi/4} - \int_0^{\pi/4}\tan(x) = 1\cdot\frac{\pi}{4} - (-\ln(\cos(x)))\Big|_0^{\pi/4}$$

$$= \frac{\pi}{4} - \left(-\ln\left(\frac{\sqrt{2}}{2}\right)+\ln(1)\right) = \underline{\underline{\frac{\pi}{4}+\ln\left(\frac{\sqrt{2}}{2}\right)}}$$

$(0+2)+(1-1$

0

g) $\int_1^e (\ln x)^2\,dx$ $\qquad g(x)=\ln(x) \Rightarrow g'(x)=\frac{1}{x}$

$\qquad f'(x)=\ln(x) \Rightarrow f(x)=x(\ln(x)-1)$

$$= (x\cdot\ln(x)-x)\cdot\ln(x)\Big|_1^e - \int_1^e (x\cdot\ln(x)-x)\cdot\frac{1}{x}\,dx$$

$$= 0-0-\underbrace{\int_1^e \ln(x)-1\,dx}_{x\cdot\ln(x)-x-x\big|_1^e} = 0-(e+2) = \underline{\underline{2-e}}$$

$f=e-2$

$(-e-(-2))$

h) $\int x \cdot \sin(2x) \, dx$ $\quad g(x) = x \Rightarrow g'(x) = 1$

$f'(x) = \sin(2x) \Rightarrow -\frac{1}{2}\cos(2x)$

$= -\frac{1}{2}\cos(2x) \cdot x + c_1 + \frac{1}{2}\int \cos(2x) = -\frac{1}{2}x\cos(2x) + c_1 + \frac{1}{2} \cdot \frac{1}{2}\sin(2x) + c_2$

$= -\frac{1}{2}x \cdot \cos(2x) + \frac{1}{4}\sin(2x) + C$

i) $\int x \cdot (1+2x)^{\frac{1}{2}} \, dx$ $\quad g(x) = x \Rightarrow g'(x) = 1$

$f'(x) = (1+2x)^{\frac{1}{2}} \Rightarrow f(x) = \frac{2}{3} \cdot \frac{1}{2} \cdot (1+2x)^{3/2} = \frac{1}{3}(1+2x)^{3/2}$

$= \frac{1}{3}(1+2x)^{3/2} \cdot x - \frac{1}{3}\int (1+2x)^{3/2} = \frac{1}{3}(1+2x)^{3/2} - \underbrace{\frac{1}{3} \cdot \frac{1}{2} \cdot \frac{2}{5}}_{1/15}(1+2x)^{5/2} + C$

$= \frac{1}{3}(1+2x)^{3/2} - \frac{1}{15}(1+2x)^{5/2} + C$

$= \frac{1}{3}(1+2x)^{3/2}\left(1 - \frac{1}{5}(1+2x)\right) + C$

j) $\int x^2 e^{\frac{1}{2}x} \, dx$ $\quad g(x) = x^2 \Rightarrow g'(x) = 2x$

$f'(x) = e^{\frac{1}{2}x} \Rightarrow f(x) = 2 \cdot e^{\frac{1}{2}x}$

$= 2 \cdot e^{\frac{1}{2}x} \cdot x^2 - \int 2x \cdot 2 \cdot e^{\frac{1}{2}x} = 2e^{\frac{1}{2}x} \cdot x^2 - 4\int \underbrace{x \cdot e^{\frac{1}{2}x}} + C$

$a(x) = x \Rightarrow a'(x) = 1$

$b'(x) = e^{\frac{1}{2}x} \Rightarrow b(x) = 2 \cdot e^{\frac{1}{2}x}$

$= 2 \cdot e^{\frac{1}{2}x} \cdot x^2 - 4\left(2 \cdot e^{\frac{1}{2}x} \cdot x - \int 2 \cdot e^{\frac{1}{2}x}\right) + C$

$= 2 \cdot e^{\frac{1}{2}x} \cdot x^2 - 4\left(2 \cdot e^{\frac{1}{2}x} \cdot x - 2 \cdot 2 \cdot e^{\frac{1}{2}x}\right) + C = 2e^{\frac{1}{2}x} \cdot x^2 - 8e^{\frac{1}{2}}x + 16e^{\frac{1}{2}x} + C$

$= e^{\frac{1}{2}x}(2x^2 - 8x + 16) + C$

k) $\int \frac{\tan(x)}{\cos^2 x}\, dx$ $\qquad g(x) = \tan(x) \Rightarrow g'(x) = \frac{1}{\cos^2(x)}$

$f'(x) = \frac{1}{\cos^2(x)} \Rightarrow f(x) = \tan(x)$

$$\int \frac{\tan(x)}{\cos^2(x)}\, dx = \tan^2(x) + C_1 - \int \frac{\tan(x)}{\cos^2(x)}\, dx$$

$$\Leftrightarrow 2 \cdot \int \frac{\tan(x)}{\cos^2(x)}\, dx = \tan^2(x) + C_1$$

$$\Leftrightarrow \int \frac{\tan(x)}{\cos^2(x)}\, dx = \underline{\underline{\frac{1}{2}\tan^2(x) + C}}$$

l) $\int \sin(2x) \cdot \cos(\frac{1}{2}x)\, dx$ $\qquad g(x) = \sin(2x) \Rightarrow g'(x) = 2\cos(2x)$

$f'(x) = \cos(\frac{1}{2}x) \Rightarrow f(x) = 2 \cdot \sin(\frac{1}{2}x)$

$$= 2 \cdot \sin(\frac{1}{2}x) \cdot \sin(2x) + C_1 - 4\int \sin(\frac{1}{2}x) \cdot \cos(2x)\, dx$$

$g(x) = \sin(\frac{1}{2}x) \Rightarrow g'(x) = \frac{1}{2}\cos(\frac{1}{2}x)$

$f'(x) = \cos(2x) \Rightarrow \frac{1}{2} \cdot \sin(2x)$

$$= 2 \cdot \sin(\frac{1}{2}x) \cdot \sin(2x) + C_1 - 4 \left(\frac{1}{2}\sin(2x) \cdot \sin(\frac{1}{2}x) + \frac{1}{4}\underbrace{\int \cos(\frac{1}{2}x) \cdot \sin(2x)\, dx}\right.$$

$$\Rightarrow \int \sin(2x) \cdot \cos(\frac{1}{2}x)\, dx = \underbrace{2 \cdot \sin(\frac{1}{2}x) \cdot \sin(2x)} + C_1 - 2\sin(2x) \cdot \sin(\frac{1}{2}x) -$$

$$\Leftrightarrow 2 \int \sin(2x) \cdot \cos(\frac{1}{2}x)\, dx = 0$$

$$\Leftrightarrow \int \sin(2x) \cdot \cos(\frac{1}{2}x)\, dx = \underline{\underline{0}}$$

2. a) $\int_0^1 (1-2x)^2 \, dx \qquad g(x) = (1-2x) \Rightarrow g'(x) = 2 = \frac{dg}{dx}$

$\Leftrightarrow dx = \frac{dg}{-2}$

$\Leftrightarrow \int_1^{-1} g^2 \cdot \frac{dg}{2} = \int_1^{-1} -\frac{g^2}{2} \, dg = -\frac{1}{6} g^3 \Big|_1^{-1} = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \underline{\underline{\frac{1}{3}}}$

b) $\int_0^2 2\left(\frac{1}{2} - t\right)^3 dt \qquad g(x) = \left(\frac{1}{2} - t\right) \Rightarrow g'(x) = -1 = \frac{dg}{dt}$

$\Leftrightarrow dt = (-1) \, dg$

$= 2\int_{1/2}^{-3/2} g^3 \cdot (-1) \, dg = -2 \cdot \left(\frac{g^4}{4} \Big|_{1/2}^{-3/2} \right) = -2 \left(\frac{81}{16} \cdot \frac{1}{4} - \frac{1}{16} \cdot \frac{1}{4} \right)$

$= -2 \cdot \frac{1}{4} \left(\frac{81}{16} - \frac{1}{16} \right) = -\frac{1}{2} \left(\frac{80}{16} \right) = -\frac{1}{2} \left(\frac{10}{2} \right) = -\frac{1}{2} \cdot 5 = \underline{\underline{-2.5}}$

c) $\int_0^4 (4-r)^{\frac{1}{2}} \, dr \qquad g(x) = 4 - r \Rightarrow g'(x) = -1 = \frac{dg}{dr}$

$\Leftrightarrow dr = -dg$

$= (-1)\int_4^0 g^{\frac{1}{2}} \, dg = (-1) \cdot \frac{2}{3} g^{\frac{3}{2}} \Big|_4^0 = -1 \cdot \left(0 - \frac{2}{3} \cdot 4^{\frac{3}{2}} \right)$

$= (-1)\left(-\frac{2}{3} \cdot \sqrt[2]{4^3} \right)$

$= \frac{2}{3} \cdot 8 = \frac{16}{3} = \underline{\underline{5 + \frac{1}{3}}}$

d) $\int_a^b (px+q)^{\frac{1}{2}}\, dx$ $\quad g(x) = px+q \Rightarrow g'(x) = p = \frac{dg}{dx}$

$\Rightarrow dx = \frac{dg}{p}$

$= \frac{1}{p}\int_{pa+q}^{pb+q} g^{\frac{1}{2}}\, dg = \left(\frac{1}{p}\right) \cdot \frac{2}{3} g^{3/2} \Big|_{pa+q}^{pb+q} = \underline{\underline{\frac{2}{3p}\left((pb+q)^{3/2} - (pa+q)^{3/2}\right)}}$

e) $\int_{-1}^{1} 3e^{-4x}\, dx$ $\quad g(x) = -4x \Rightarrow g'(x) = -4 = \frac{dg}{dx}$

$\Leftrightarrow dx = \frac{dg}{-4}$

$= \int_{4}^{-4} 3e^{g} \cdot \left(-\frac{1}{4}\right) dg = -\frac{3}{4}\int_{4}^{-4} e^{g}\, dg = -\frac{3}{4} \cdot e^{g} \Big|_{4}^{-4}$

$= \underline{\underline{-\frac{3}{4}\left(e^{-4} - e^{4}\right)}}$

f) $\int_{1}^{\ln 2} \sqrt{e^{-x}}\, dx$ $\quad g(x) = -x \Rightarrow g'(x) = -1 = \frac{dg}{dx}$

$\Leftrightarrow dx = dg \cdot (-1)$

$= (-1)\int_{-1}^{-\ln 2} e^{\frac{g}{2}}\, dg = (-1) \cdot \left(2 \cdot e^{\frac{g}{2}} \Big|_{-1}^{-\ln 2}\right) = \underline{\underline{-2\left(e^{-\frac{\ln 2}{2}} - e^{-\frac{1}{2}}\right)}}$

3\. a) $\int_1^2 \sqrt{e^{3x} + e^{2x}}$

$2 = \ln(t)$

$\Leftrightarrow \log_e(t) = 2 \Rightarrow t = e^2$

$\frac{dx}{dt} = \frac{1}{t} \Leftrightarrow dx = \frac{dt}{t}$

$= \int_{e^1}^{e^2} \sqrt{e^{3\ln(t)} + e^{2\ln(t)}} \cdot \frac{dt}{t}$

$= \int_{e^1}^{e^2} \underbrace{\sqrt{t^3 + t^2}}_{\sqrt{t^2(t+1)}} \cdot \frac{dt}{t} = \int_{e}^{e^2} t\sqrt{t+1} \; \frac{dt}{t}$

$= \int_{e}^{e^2} \sqrt{t+1} \; dt \Rightarrow \int_{e+1}^{e^2+1} g^{\frac{1}{2}} \; dg = \frac{2}{3} g^{3/2} \Big|_{e+1}^{e^2+1}$

$g(t) = t+1$

$\Leftrightarrow g'(t) = 1$

$\Leftrightarrow dg = dt$

$= \underline{\underline{\frac{2}{3}(e^2+1)^{3/2} - \frac{2}{3}(e+1)^{3/2}}}$

b) $\int_0^{\frac{1}{2}\sqrt{2}} \frac{1}{\sqrt{1-x^2}} \; dx$

$\frac{1}{2}\sqrt{2} = \sin(t) \Rightarrow t = \pi/4$

$0 = \sin(t) \Rightarrow 0$

$\Rightarrow \frac{dx}{dt} = \cos(t) \Leftrightarrow dx = -\cos(t) \cdot dt$

$= \int_0^{\pi/4} \frac{1}{\sqrt{\cos^2(t)}} \cdot (-\cos(t)) \; dt = \int_0^{\pi/4} = 1 \; dt = x \Big|_0^{\pi/4} = \underline{\underline{\pi/4}}$

4.a) $a(x) = -\frac{3}{5} \cdot x^8$

$\Rightarrow A(x) = -\frac{3}{5} \cdot x^9 \cdot \frac{1}{9} = -\frac{3}{45} x^9 = -\frac{1}{15} x^9$

b) $h(u) = (u^{10} - 6u^6 + 9u^2) \cdot 5u^4 = 5u^{14} - 30u^{10} + 45u^6$

$\Rightarrow H(u) = \frac{1}{3} u^{15} - \frac{30}{11} u^{11} + \frac{45}{7} u^7$

c) $r(s) = \frac{3t^4 - 3t^2 + 5t - 7}{4t^2} \cdot s$

d) $g(t) = x^2 \cdot e^{2x}$

$g(t) = x^2 \Rightarrow g'(x) = 2x$
$f'(x) = e^{2x} \Rightarrow F(x) = \frac{1}{2} e^{2x}$

$G(x) = \frac{1}{2} e^{2x} \cdot x^2 - \int \frac{1}{2} e^{2x} \cdot 2x \, dx$

$\Rightarrow g'(x) = 2$ $\quad$ $\left(f'(x) = \frac{1}{2} e^{2x}, \; g(x) = 2x \right)$
$F(x) = \frac{1}{4} e^{2x}$

$= \frac{1}{2} e^{2x} \cdot x^2 - \left(2x \cdot \frac{1}{4} e^{2x} - \int 2 \cdot \frac{1}{4} e^{2x} dx \right)$

$= \frac{1}{2} e^{2x} \cdot x^2 - \left(2x \cdot \frac{1}{4} e^{2x} - \frac{1}{4} e^{2x} \right)$

$= \frac{1}{2} e^{2x} x^2 - 2x \cdot \frac{1}{4} e^{2x} + \frac{1}{4} e^{2x} = \frac{1}{2} e^{2x} \left(x^2 - x + \frac{1}{2} \right)$

e) $r(a) = s \cdot \sin(a) \cdot \cos(a)$

$a'(x) = s \cdot x \Rightarrow a(x) = \frac{sx^2}{2}$
$i = \sin(x)$
$i' = \cos(x)$

$R(a) = \frac{1}{2} s \cdot \sin^2(a)$

f) $(3x-5)^6$ $\quad$ Substitution $\Rightarrow$ $g(x) = 3x - 5 \Rightarrow g'(x) = 3 = \frac{dg}{dx}$

$\Leftrightarrow dx = \frac{dg}{3}$

$\Rightarrow \int g^6 \cdot \frac{dg}{3} = \int \frac{g^6}{3} dg = \frac{1}{7} \cdot \frac{1}{3} g^7 = \frac{1}{21} g^7 = \frac{1}{21} (3x-5)^7$

g) $h(t) = (2t-1)^{\frac{1}{2}}$ $\Rightarrow$ $a'(x) = x^{\frac{1}{2}} \Rightarrow a(x) = \frac{2}{3} \cdot x^{3/2}$ ⑤

$i = 2t-1 \Rightarrow i' = 2$

$H(t) = \frac{1}{2} \cdot \frac{2}{3} \cdot (2t-1)^{3/2} = \underline{\underline{\frac{1}{3}(2t-1)^{3/2}}}$

h) $f(t) = \frac{t^n}{n!}$

$F(t) = \frac{1}{(n+1)} \cdot t^{n+1} \cdot \frac{1}{n!} = \underline{\underline{\frac{1}{(n+1)!} \cdot t^{n+1}}}$

i) $r(x) = e^{\sin x} \cdot \cos x$ Subst. $g(x) = \sin(x) \Rightarrow g'(x) = \cos(x) = \frac{dg}{dx}$

$\Leftrightarrow dx = \frac{dg}{\cos(x)}$

$\int e^g \cdot \cancel{\cos(x)} \cdot \frac{dg}{\cancel{\cos(x)}} = \int e^g \, dg = e^g = \underline{\underline{e^{\sin(x)}}}$

j) $g(x) = \frac{e^x}{2} + \frac{e^{-x}}{2}$

$g(x) = \frac{e^x}{2} + (-1) \cdot \frac{e^{-x}}{2} = \underline{\underline{\frac{e^x - e^{-x}}{2}}}$

k) * *

---

l) $b(t) = t \cdot e^{-t}$, $g(x) = t \Rightarrow g'(x) = 1$

$f'(x) = e^{-t} \Rightarrow f(x) = (-1) \cdot e^{-t}$

$B(t) = -e^{-t} \cdot t - \int -e^{-t} \cdot 1 \, dt$

$= -e^{-t} \cdot t + \int e^{-t} \, dt = -e^{-t} \cdot t + (-1) e^{-t} = -e^{-t} \cdot t - e^{-t}$

$= \underline{\underline{e^{-t} \cdot (-t-1)}}$

m) $c(x) = x \cdot \ln(x)$ $g(x) = \ln(x) \Rightarrow g'(x) = \frac{1}{x}$

$f'(x) = x \Rightarrow f(x) = \frac{x^2}{2}$

$C(x) = \frac{x^2}{2} \cdot \ln(x) - \int \frac{x^2}{2} \cdot \frac{1}{x} dx = \frac{x^2}{2} \ln(x) - \int \frac{x}{2} \, dx$

$= \frac{1}{2}x^2 \cdot \ln(x) - \frac{1}{4}x^2 = \underline{\underline{\frac{1}{2}x^2 \left(\ln(x) - \frac{1}{2}\right)}}$

n) $f(s) = \frac{\ln(s)}{s^2} = \ln(s) \cdot s^{-2}$

$g(s) = \ln(s) \Rightarrow g'(s) = \frac{1}{s}$

$f'(x) = s^{-2} \Rightarrow -s^{-1}$

$F(s) = -s^{-1} \cdot \ln(s) - \int \frac{1}{s} \cdot (-1) \cdot \frac{1}{s} \, ds$

$= -\frac{1}{s} \cdot \ln(s) - \int -\frac{1}{s^2} \, ds = -\frac{1}{s} \cdot \ln(s) + \int s^{-2} \, ds$

$= -\frac{1}{s} \cdot \ln(s) + -s^{-1}$

$= -\frac{1}{s} \cdot \ln(s) - \frac{1}{s} = -\frac{1}{s}(\ln(s) + 1)$

o) $j(r) = \sin(r) \cdot \cos(r)$

① $\begin{cases} a'(r) = r \Rightarrow a(r) = \frac{1}{2} r^2 \\ i(r) = \sin(r) \Rightarrow i'(r) = \cos(r) \end{cases}$

① $J(r) = \frac{1}{2} \sin^2(r)$

oder

② $\begin{cases} a'(r) = r \Rightarrow a'(r) = \frac{1}{2} r^2 \\ i(r) = \cos(r) \Rightarrow i'(r) = -\sin(r) \end{cases}$

② $J(r) \; -\frac{1}{2} \cos^2(r)$

---

5. a) $\int_1^3 -3u^{-4} \, du = -3 \int_1^3 u^{-4} \, du = -3 \cdot \left(-\frac{1}{3} \frac{1}{u^3} \Big|_1^3 \right)$

$= -3 \left(-\frac{1}{3} \cdot \frac{1}{27} + \frac{1}{3} \frac{1}{1} \right) = -3 \left(-\frac{1}{81} + \frac{1}{3} \right) = -3 \left(-\frac{1}{81} + \frac{27}{81} \right) = -3 \cdot \frac{26}{81} = \frac{-78}{81}$

$= -\frac{26}{27}$

5. b) $\int_0^{\frac{\pi}{2\omega}} \cos(\omega t)\, dt$ $\qquad a'(x) = \cos(x) \Rightarrow a(x) = \sin(x)$

$\qquad\qquad\qquad\qquad\qquad\qquad i(x) = \omega t \Rightarrow i'(x) = \omega$

$$= \frac{1}{\omega} \sin(\omega t) \Big|_0^{\frac{\pi}{2\omega}} = \frac{1}{\omega} \sin\left(\frac{\pi \cancel{\omega}}{2 \cancel{\omega}}\right) - \underbrace{\frac{1}{\omega} \sin(0)}_{=0}$$

$$= \frac{1}{\omega} \cdot \underbrace{\sin\left(\frac{\pi}{2}\right)}_{1} = \underline{\underline{\frac{1}{\omega}}}$$

c) $\int_0^{\pi/4} \sin^2(x) \cdot \cos(x)\, dx$ $\qquad g(x) = \sin(x) \Rightarrow g'(x) = \cos(x) = \frac{dg}{dx}$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \Leftrightarrow dx = \frac{dg}{\cos(x)}$

$$\Leftrightarrow \int_0^{\frac{\sqrt{2}}{2}} g^2 \cdot \cancel{\cos(x)} \cdot \frac{dg}{\cancel{\cos(x)}} = \int_0^{\frac{\sqrt{2}}{2}} g^2\, dg = \frac{1}{3} g^3 \Big|_0^{\frac{\sqrt{2}}{2}}$$

$$= \frac{1}{3} \left(\frac{\sqrt{2} \cdot \sqrt{2} \cdot \sqrt{2}}{2 \cdot 2 \cdot 2} \right) = \frac{1}{3} \left(\frac{2 \cdot \sqrt{2}}{8} \right) = \frac{1}{3} \left(\frac{2^{\frac{2}{2}} \cdot 2^{\frac{1}{2}}}{8} \right)$$

$$= \frac{1}{3} \left(\frac{2^{3/2}}{8} \right) = \underline{\underline{\frac{2^{3/2}}{24}}} = \frac{2^{-3/2}}{3} \;?$$

$\frac{3 \cdot 8}{3 \cdot 2^3} \qquad 2^{3/2 - 3} = 2^{-3/2}$

$2^{3/2}/3$