Analysis-Aufgaben: Integralrechnung 3

① a) $\int_0^1 (t^2+1)e^t\,dt \quad = e^t(t^2+1)\Big|_0^1 - \underbrace{\int_0^1 2te^t\,dt}$

Wähle $f(t)=t^2+1 \Rightarrow f'(t)=2t$

$g'(t)=e^t \Rightarrow g(t)=e^t$

Wähle $a(t)=2t \Rightarrow a'(t)=2$

$b'(t)=e^t \Rightarrow b(t)=e^t$

$$= e^t(t^2+1)\Big|_0^1 - \left(2te^t\Big|_0^1 - \int_0^1 2e^t\,dt\right)$$

$$= e^t(t^2+1)\Big|_0^1 - 2te^t\Big|_0^1 + 2e^t\Big|_0^1 = \underline{\underline{2e-3}}$$

b) $\int_{\pi/4}^{\pi/3} \frac{\cos\varphi}{1+2\cdot\sin\varphi}\,d\varphi \quad = \int_{1+2\cdot\frac{\sqrt{2}}{2}}^{1+2\cdot\frac{\sqrt{3}}{2}} \frac{\cos\varphi}{g}\cdot\frac{dg}{2\cdot\cos\varphi}$

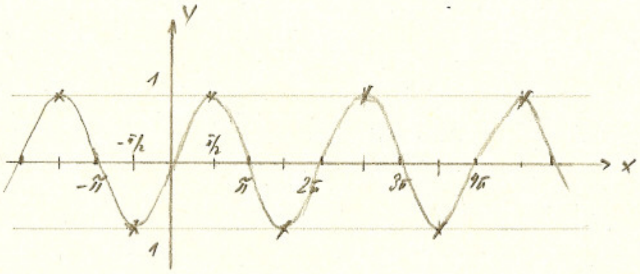
Setze $g(\varphi)=1+2\cdot\sin\varphi$

$\Rightarrow \frac{dg}{d\varphi}=2\cos\varphi$

$\Leftrightarrow d\varphi=\frac{dg}{2\cos\varphi}$

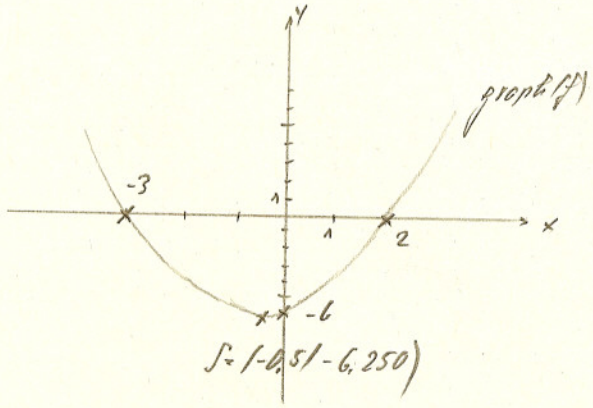
$$= \frac{1}{2}\cdot\int_{\ldots}^{\ldots}\frac{1}{g}\,dg = \underline{\underline{0{,}062}}$$

② a)



b) $\underline{\underline{A}} = \int_0^{\pi} f(x)\,dx + \left|\int_{\pi}^{2\pi} f(x)\,dx\right| = \underline{\underline{4}}$

③ a)



b) $\underline{\underline{A}} = \left|\int_{-2}^{2} f(x)\,dx\right| + \int_{2}^{3} f(x)\,dx = \underline{\underline{21{,}5}}$

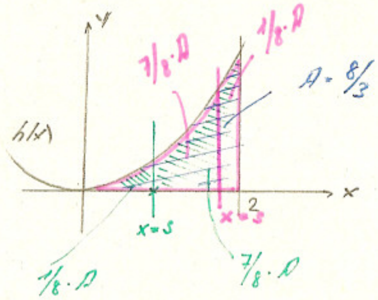
c) $\underline{\underline{x_1=-3{,}881}}$, $\underline{\underline{x_2=-2}}$, $\underline{\underline{x_3=4{,}381}}$

d) Inhalt der Fläche zwischen graph(f) und der x-Achse ist über

- $[x_1,-3]$ und $[-3,-2]$ gleich
- $[-2,2]$ und $[2,x_3]$ gleich

④ a) $\int_0^2 h(x)\,dx = \underline{\underline{2{,}667}}$

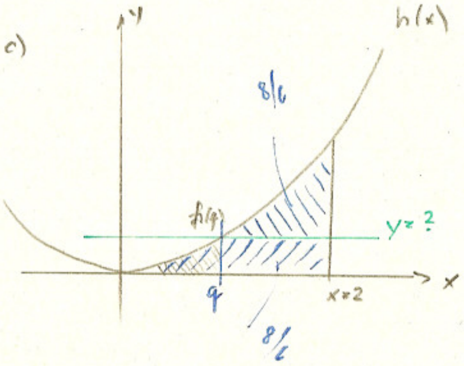
b)



1. Fall. $\int_0^s h(x)\,dx = \frac{1}{8}\cdot A = \frac{1}{3} \Rightarrow \underline{\underline{s_1 = 1}}$

2. Fall. $\int_0^s h(x)\,dx = \frac{7}{8}\cdot A = \frac{7}{3} \rightarrow \underline{\underline{s_2 = 1{,}913}}$

c)



$\cdot \int_0^q h(x)\,dx + (2-q)\cdot h(q) = \frac{8}{6} \Rightarrow$

$q_1 = 2{,}732\,,\ \underline{q_2 = 1}\,;\ q_3 = -0{,}732$

$\Rightarrow \underline{\underline{y}} = h(q_2) = \underline{\underline{1}}$

Weiterer Lösungsansatz: $\cdot \int_q^2 h(x) - q^2\,dx$