Analysis-Aufgaben: Integralrechnung 4

1 a) $\int_1^4 \frac{1-x}{\sqrt{x}} \cdot dx = \int_1^4 x^{-1/2} - x^{1/2}\, dx$

$= 2 \cdot x^{1/2} - \frac{2}{3} \cdot x^{3/2} \Big|_1^4 = \underline{\underline{-\frac{8}{3}}}$

b) $\int \alpha^3 \cdot \cos(\alpha^2)\, d\alpha = \text{"} \int \alpha \cdot u \cdot \cos u \cdot \frac{du}{2\alpha} \text{"} = \frac{1}{2} \cdot \int u \cos u \, du$

Setze $u := \alpha^2 \Rightarrow d\alpha = \frac{du}{2\alpha}$

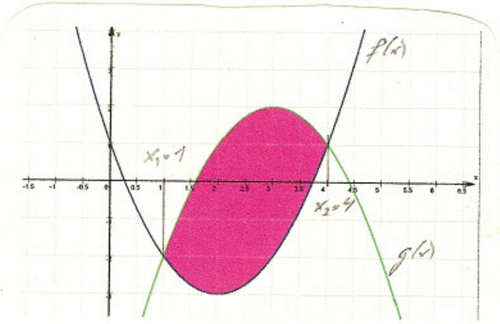
Wähle $f(u) = u \Rightarrow f'(u) = 1$
$g'(u) = \cos u \Rightarrow g(u) = \sin u$

$= \frac{1}{2} \cdot \left(u \cdot \sin u + C_1 - \int \sin u \, du \right)$

$= \frac{1}{2} \cdot \left(u \cdot \sin u + C_1 - (-\cos u) + C_2 \right)$

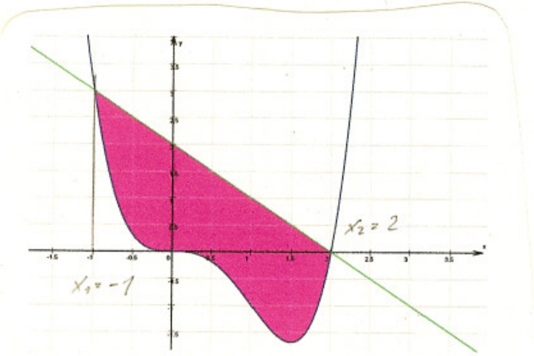
$= \frac{1}{2} \cdot (u \cdot \sin u + \cos u) + C = \underline{\underline{\frac{1}{2} \cdot (\alpha^2 \cdot \sin \alpha^2 + \cos \alpha^2) + C}}$

2



$\underline{A} = \int_1^4 g(x) - f(x)\, dx = \underline{\underline{9}}$

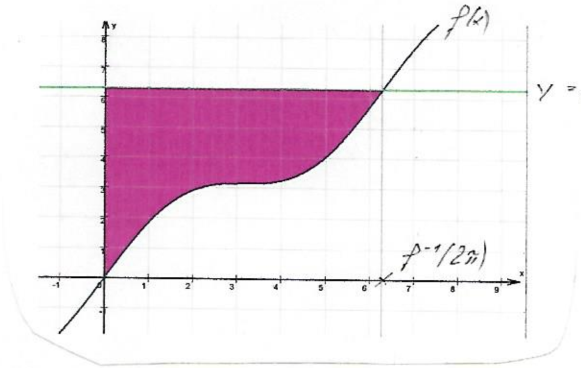
3



$\underline{A} = \int_{-1}^{2} g(x) - f(x)\, dx = \underline{\underline{5{,}4}}$

④

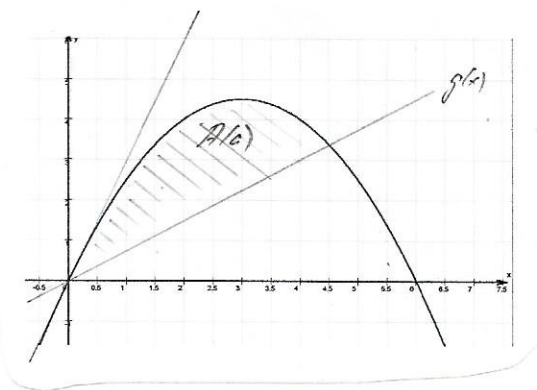
a)



b) i) $\underline{\underline{0{,}960}}$, ii) $\underline{\underline{6{,}935}}$, iii) $\underline{\underline{18{,}651}}$

c) $\underline{\underline{A}} = 2\pi \cdot f^{-1}(2\pi) - \int_0^{f^{-1}(2\pi)} f(x)\,dx \quad = \underline{\underline{19{,}739}}$

⑤



a) $\underline{\underline{A}} = \int_0^6 f(x)\,dx = \underline{\underline{18}}$

b) $f(x) = g(x) \quad \Rightarrow \quad x_1 = 0 , \ x_2 = -2a + 6$

$\Rightarrow \underline{\underline{S_1 = (0/0)}} , \ \underline{\underline{S_2 = (-2a+6 / -2a^2 + 6a)}}$

c) $\int_0^{-2a+6} f(x) - g(x)\,dx = \underline{\underline{-\tfrac{2}{3} \cdot (a-3)^3}} = \underline{\underline{A(a)}}$

d) für $a \neq f'(0) \iff \underline{\underline{a \neq 3}}$

e) $A(a) \overset{c)}{=} -\tfrac{2}{3} \cdot (a-3)^3 \overset{a)}{=} 8 \cdot 18 = 144 \quad \iff \quad \underline{\underline{a = -3}}$

⑥ f = Parabel $\Rightarrow$ $f(x) = ax^2 + bx + c$

$P = (0|0) \in graph(f)$ $\Rightarrow$ $c \overset{(*)}{=} 0$

$Q = (4|0) \in graph(f)$ $\Rightarrow$ $16a + 4b = 0$

$\int_0^4 f(x)\,dx = \frac{64}{3}$ $\Leftrightarrow$ $\frac{1}{3} \cdot ax^3 + \frac{1}{2} \cdot bx^2 + cx + d \Big|_0^4 = \frac{64}{3}$

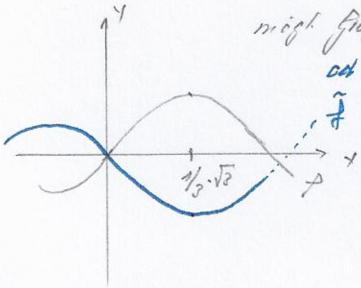
$\overset{(*)}{\Leftrightarrow}$ $\frac{1}{3} \cdot a \cdot 64 + \frac{1}{2} \cdot b \cdot 16 = \frac{64}{3}$

$\Rightarrow$ $a = -2,\ b = 8$

$\Rightarrow$ $f(x) = -2x^2 + 8x$

⑦ $A = \left| \int_{-3}^{2} f(x)\,dx \right| = 52{,}083$

⑧ $f(x)$ = Polynom fkt 3 Grades $\Rightarrow$ $f(x) = ax^3 + bx^2 + cx + d$



Ursprung = WP $\Rightarrow$ · $f(0) = 0$ $\overset{(*)}{\Rightarrow}$ $d = 0$

· $f''(0) = 0$ $\overset{(*)(**)}{\Rightarrow}$ $b = 0$

"Steigung" $\Rightarrow$ $f'(\frac{1}{3}\cdot\sqrt{3}) = 0$ $\overset{(**)}{\Rightarrow}$ $a + c = 0$

"Fläche" $\Rightarrow$ $\left| \int_0^q f(x)\,dx \right| = \frac{3}{4}$ $\Leftrightarrow$ $\frac{1}{4} ax^4 + \frac{1}{3} bx^3 + \frac{1}{2} cx^2 + dx + e \Big|_0^q = \frac{3}{4}$

$\underset{(**)}{\overset{(*)}{\Leftrightarrow}}$ $\frac{1}{4} aq^4 + \frac{1}{2} cq^2 = \frac{3}{4}$

$f(q) = 0$ $\underset{(**)}{\overset{(*)}{\Leftrightarrow}}$ $aq^3 + cq = 0$ $\Rightarrow$ $q_1 = 0$

$q_{2,3} = \pm\sqrt{-\frac{c}{a}}$ $\overset{**}{\Rightarrow}$ $q = \pm 1$

$\Rightarrow$ $\frac{1}{4} a + \frac{1}{2} c = \frac{3}{4}$

$\Rightarrow$ $a = -3,\ c = 3$

$\Rightarrow$ $f(x) = -3x^3 + 3x$

⑨ $g(x) = \frac{1}{3}x^3 - x$

a) Vollständige Diskussion: Typ: Polynom, 3. Grades

· NS. $g(x) \overset{!}{=} 0 \iff \frac{1}{3}x^3 - x = 0$

$\iff x(\frac{1}{3}x^2 - 1) = 0 \implies \underline{\underline{x_1 = 0}}$

$\underline{\underline{x_{2,3} = \pm\sqrt{3}}}$

· ES. 'mögl. ES' $\iff g'(x) \overset{!}{=} 0$

$\iff x^2 - 1 = 0 \implies \underline{x_{4,5} = \pm 1}$

$g''(x) = 2x \implies g''(x_4) = 2 \neq 0 \implies \underline{\underline{x_4 = 1 \text{ ist ES}}}$

$\underline{\underline{\text{mit Pkt. Min.} = -\frac{2}{3}}}$

$g''(x_5) = -2 \neq 0 \implies \underline{\underline{x_5 = -1 \text{ ist ES}}}$

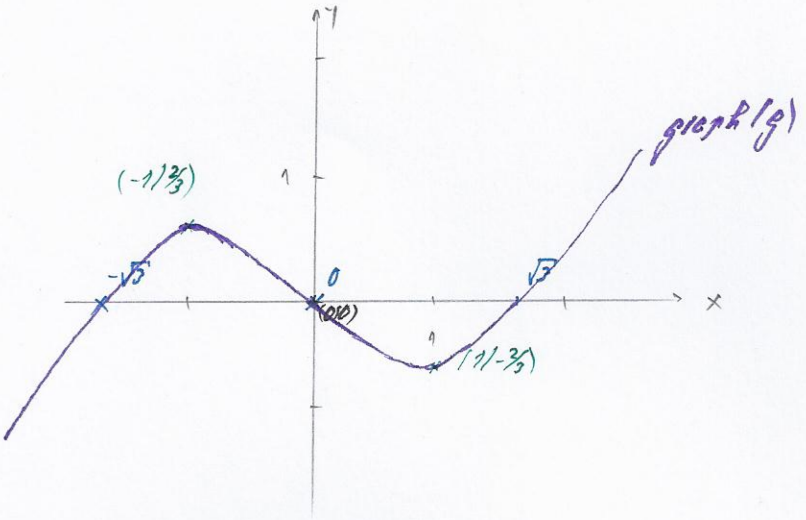
$\underline{\underline{\text{mit Pkt. Max.} = \frac{2}{3}}}$

· WP 'mögl. WP' $\iff g''(x) \overset{!}{=} 0$

$\iff x_6 = 0$

$g'''(x) = 2 \neq 0, \forall x \in D(g)$

$\implies \underline{\underline{WP = (0|0)}}$



b) $T(x) = ax + b$

parallel zu $y = 3x + 6$

$\implies T(x) = 3x + b$

Tangente $\implies g'(x) = 3$

$\iff x^2 - 1 = 3 \implies \underline{x_{7,8} = \pm 2}$ (Stellen für die Berührungspunkte)

$g(x_7) = T(x_7) \iff \frac{2}{3} = 6 + b \implies \underline{b = -\frac{16}{3}}$

$\implies$ 1. Berührungspunkt $P_1 = (2|\frac{2}{3})$ mit $T_1(x) = 3x - \frac{16}{3}$

$g(x_8) = T(x_8) \implies$ 2. Berührungspunkt $P_2 = (-2|-\frac{2}{3})$, mit $T_2(x) = 3x + \frac{16}{3}$

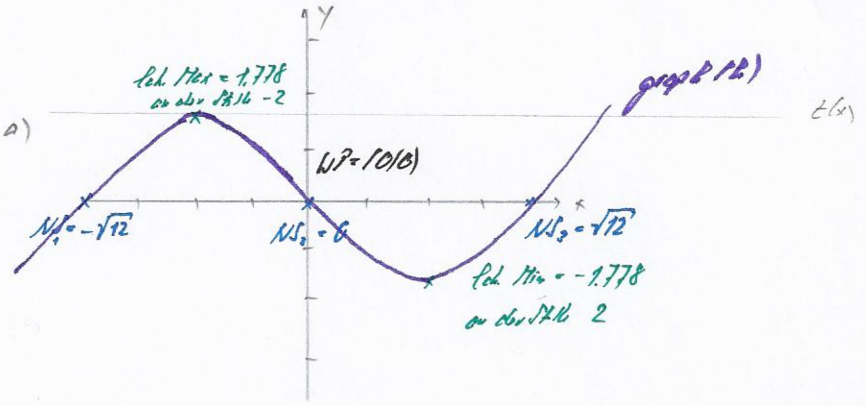
c) $T_1(x) = g(x) \Rightarrow \underline{\underline{S_1 = (-4 / -\frac{52}{3})}}$ $\qquad$ $T_2(x) = g(x) \Rightarrow \underline{\underline{S_2 = (4 / \frac{52}{3})}}$

d) $\underline{\underline{A_1}} = \int_{-4}^{2} g(x) - T_1(x)\,dx$ $\qquad$ $\underline{\underline{A_2}} = \int_{-2}^{4} T_2(x) - g(x)\,dx$

$= \underline{\underline{36}}$ $\qquad$ $= \underline{\underline{36}}$

(10) $h(x) = 1/9 \cdot x^3 - 4/3 \cdot x$

a)



b) a) $\Rightarrow P = (-2 / 1.778) \Rightarrow x_0 = -2$

$t(x) = 1.778$

$h(x) = t(x) \Rightarrow x_1 = 4$

$\Rightarrow \underline{x_1 = -2 \cdot x_0}$

c) $\underline{\underline{A}} = \int_{-2}^{4} t(x) - g(x)\,dx = \underline{\underline{12}}$

d) $\cdot\ f(x) = ax^3 + 3x \Rightarrow f'(x) = 3ax^2 + 3 \overset{!}{=} 0$

$\Leftrightarrow \underline{x_0 = \pm\sqrt{\frac{-3}{3a}}}$ (Diese Lösg. ex. nur, falls $a \cdot 3 < 0$)

$\cdot$ Äquivalent zur Bedingung $x_1 = -2x_0$ ist $f(x_1) = f(x_0)$

$f(x_0) = a \cdot x_0^3 + 3x_0$

$f(x_1) = f(-2x_0)$

$= -8ax_0^3 - 23x_0$

$\Rightarrow ax_0^3 + 3x_0 = -8ax_0 - 23x_0$

$\Leftrightarrow 9ax_0^3 + 33x_0 = 0$

$\Leftrightarrow x_0 \cdot (9ax_0^2 + 33) = 0$

$\Rightarrow$ 1. Lösg für x_0 : $x_0 = 0$ (↯ Voraussetzung)

$\underline{\text{2. \& 3. Lösg für } x_0}$: $\pm\sqrt{\frac{-33}{9a}} = \underline{\underline{\pm\sqrt{\frac{-3}{3a}}}}$