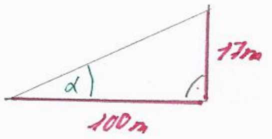
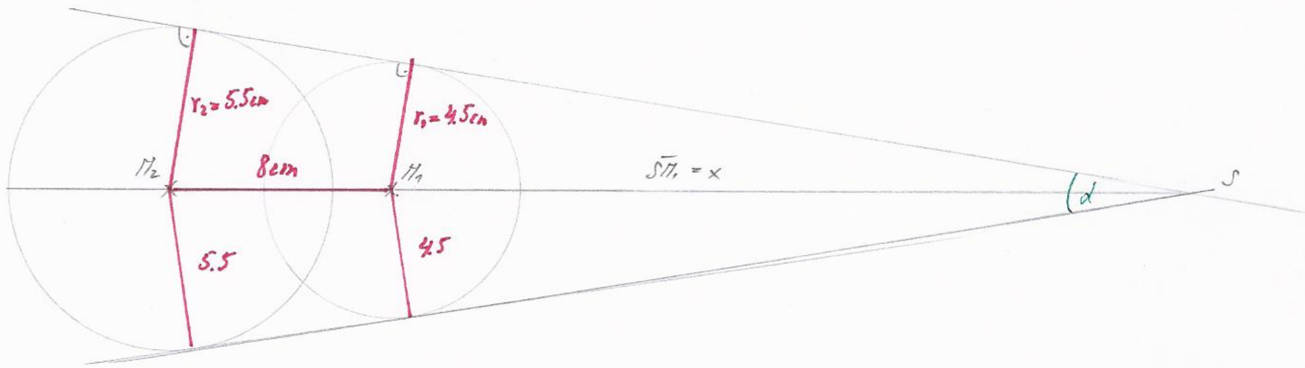
Geometrie-Aufgaben: Trigonometrie 8

(1) 17%-Steigung heißt auf 100m horizontal 17m Höhengewinn entspricht.



$$\tan\alpha = \frac{17}{100} \Rightarrow \underline{\underline{\alpha}} = \tan^{-1}\left(\frac{17}{100}\right) = \underline{\underline{9{,}648°}}$$

(2)



$$\frac{r_2}{\overline{M_1M_2} + \overline{SM_1}} = \sin\frac{\alpha}{2} \iff \overline{SM_1} = \frac{r_2 - \overline{M_1M_2}\cdot\sin\left(\frac{\alpha}{2}\right)}{\sin\frac{\alpha}{2}}$$

$$\frac{r_1}{\overline{SM_1}} = \sin\frac{\alpha}{2} \iff \overline{SM_1} = \frac{r_1}{\sin\frac{\alpha}{2}}$$

$$\Rightarrow \frac{r_2 - \overline{M_1M_2}\cdot\sin\frac{\alpha}{2}}{\sin\frac{\alpha}{2}} = \frac{r_1}{\sin\frac{\alpha}{2}}$$

$$\iff r_2 - \overline{M_1M_2}\cdot\sin\frac{\alpha}{2} = r_1$$

$$\iff \sin\frac{\alpha}{2} = \frac{r_2 - r_1}{\overline{M_1M_2}}$$

$$\Rightarrow \frac{\alpha}{2} = \sin^{-1}\left(\frac{5.5-4.5}{8}\right) = 7{,}181°$$

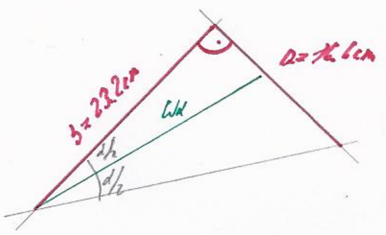
$$\Rightarrow \underline{\underline{\alpha = 14{,}362°}}$$

oder "gleichsetzen"

$$\Rightarrow \overline{SM_1} = \ldots = 36cm$$

$$\Rightarrow \frac{\alpha}{2} = \sin^{-1}\left(\frac{r_1}{x}\right) = \sin^{-1}\left(\frac{r_2}{\overline{M_1M_2}+x}\right)$$

(3)



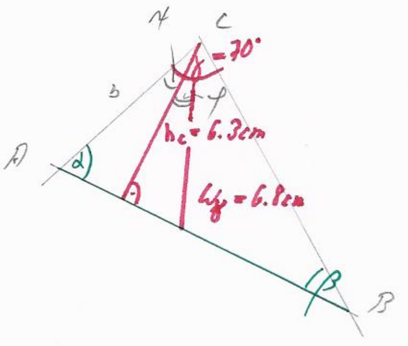
$$\tan\alpha = \frac{a}{b} \Rightarrow \underline{\alpha} = \tan^{-1}\left(\frac{a}{b}\right)$$

$$= \tan^{-1}\left(\frac{16.6}{23.2}\right) = \underline{35{,}584°}$$

$$\cos\frac{\alpha}{2} = \frac{b}{w_\alpha} \iff \underline{\underline{w_\alpha}} = \frac{b}{\cos\frac{\alpha}{2}}$$

$$= \frac{23.2}{\cos\left(\frac{35{,}584\ldots}{2}°\right)} = \underline{\underline{24{,}365cm}}$$

④



$$\cos\varphi = \frac{h_c}{w_\gamma} \Rightarrow \varphi = \cos^{-1}\left(\frac{h_c}{w_\gamma}\right)$$

$$\Rightarrow \underline{\varphi} = \cos^{-1}\left(\frac{6.3}{6.8}\right) = \underline{22{,}109°}$$

$$\Rightarrow \underline{\psi} = \gamma/2 - \varphi = \underline{12{,}891°}$$

$$\Rightarrow \underline{\underline{\alpha}} = 180° - 90° - \psi = \underline{\underline{77{,}109°}}$$

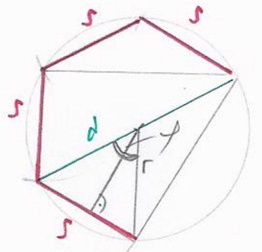
$$\Rightarrow \underline{\underline{\beta}} = 180° - (\alpha+\gamma) = \underline{\underline{32{,}891°}}$$

· $\sin\alpha = \frac{h_c}{b} \Rightarrow \underline{b} = \frac{h_c}{\sin\alpha} = \underline{6{,}463\,cm}$

· $\frac{b}{\sin\beta} = \frac{c}{\sin\gamma} \Rightarrow \underline{c} = \frac{b\cdot\sin\gamma}{\sin\beta} = \underline{\underline{11{,}183\,cm}}$

⑤ a) "gerade" Eckenzahl ⇒ längste Diagonale geht durch den Mittelpunkt des Umkreises.

⇒ · $d = 2r$

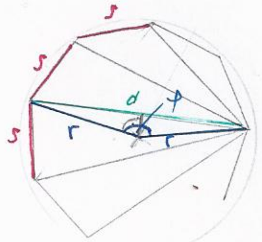


· $\frac{s/2}{r} = \sin(\varphi/2)$, mit $\varphi = \frac{360°}{n}$

$$\Rightarrow \underline{\underline{d}} = 2\cdot\underbrace{\left(\frac{s/2}{\sin\left(\frac{360°}{2n}\right)}\right)}_{=r} = \underline{\underline{\frac{s}{\sin\left(\frac{180°}{n}\right)}}}$$

b) "ungerade" Eckenzahl

(*) $\sin 2\alpha = 2\cdot\sin\alpha\cdot\cos\alpha$



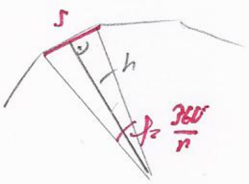
d = längste Sehne im Kreisrechteck, mit $\varphi = \frac{360°}{n}\cdot\frac{n-1}{2} = 180° - \frac{180°}{n}$

$$\frac{d/2}{r} = \sin(\varphi/2) = \sin\left(90° - \frac{90°}{n}\right) = \cos\left(\frac{90°}{n}\right)$$

$$r \overset{(a)}{=} \frac{s}{2\cdot\sin\left(\frac{180°}{n}\right)}$$

$$\Rightarrow \underline{\underline{d}} = 2\cdot\frac{s}{2\cdot\sin\left(\frac{180°}{n}\right)}\cdot\cos\left(\frac{90°}{n}\right) \overset{(*)}{=} \frac{s}{2\sin\left(\frac{90°}{n}\right)\cdot\cos\left(\frac{90°}{n}\right)}\cdot\cos\left(\frac{90°}{n}\right) = \underline{\underline{\frac{s}{2\cdot\sin\left(\frac{90°}{n}\right)}}}$$

⑥



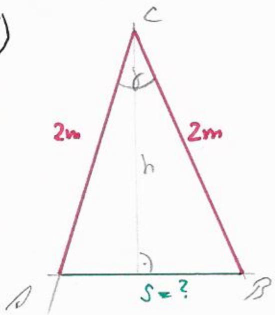
Fläche = n gleichschenklige Δ mit Basislänge = s

"Spitzenwinkel" $\varphi = \frac{360°}{n}$

$$\frac{s/2}{h} = \tan(\varphi/2) = \tan\left(\frac{180°}{n}\right) \Rightarrow h = s/2 : \tan\left(\frac{180°}{n}\right)$$

$$\Rightarrow \underline{\underline{A}} = n\cdot\frac{s}{2}\cdot\frac{s}{2\cdot\tan\left(\frac{180°}{n}\right)} = \underline{\underline{n\cdot\frac{s^2}{4\cdot\tan(\pi/n)}}}$$

⑦



$$\frac{s\cdot h}{2} = 1.5m^2 = \frac{2m\cdot 2m\cdot \sin\gamma}{2}$$

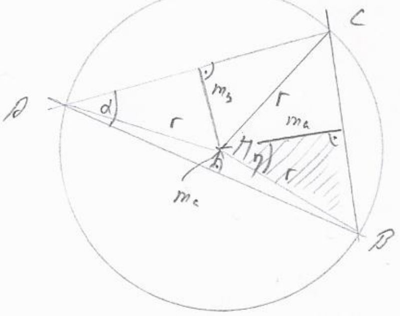
$$\Rightarrow \gamma = \sin^{-1}\left(\frac{1.5}{2}\right) = 48.590°$$

(2. Lösung: $\gamma_2 = 131,410°$ ↯ spitzwinklig)

$$\Rightarrow \alpha = \beta = \frac{180° - \gamma}{2} = 65.705°$$

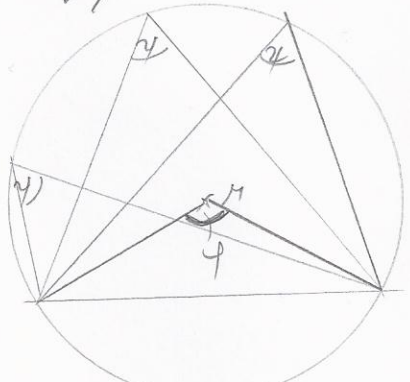
$$\frac{s/2}{2} = \cos\beta \Rightarrow s = 2\cdot 2\cos\beta = 1.646m$$

⑧



Was wir wissen müssen:

"Über jeder Sehne ist der Mittelpunktswinkel φ doppelt so groß wie jeder Umfangswinkel ψ"



$\varphi = 2\cdot\psi$

m_a, m_b, m_c: Mittelsenkrechte

$$\Rightarrow \eta = \alpha$$

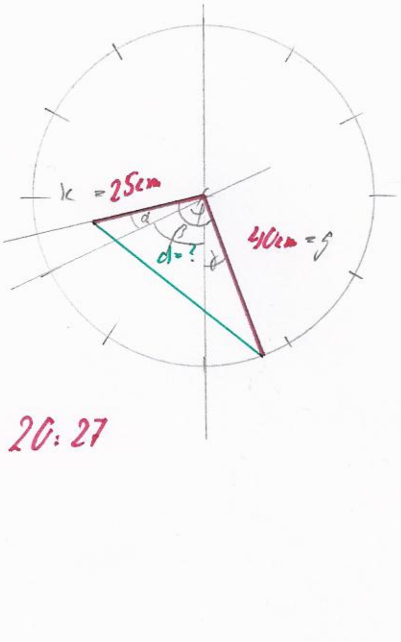
$$\Rightarrow \sin\eta = \sin\alpha = \frac{a/2}{r}$$

$$\Leftrightarrow r = \frac{a}{2\cdot\sin\alpha}$$

$$\Leftrightarrow 2r = \frac{a}{\sin\alpha}$$

□

⑨



20:27

$$\varphi = \alpha + \beta + \gamma$$

$$\alpha = \frac{360°/12}{60°}\cdot 27° = 13,5°$$

$$\beta = 360°/6 = 60°$$

$$\gamma = 180° - 360°/60 \cdot 27° = 18°$$

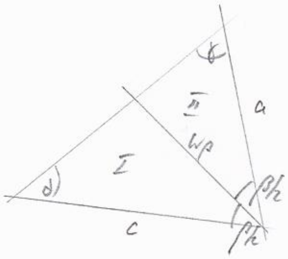
$$\Rightarrow \varphi = \ldots = 91,5°$$

$$d^2 = k^2 + g^2 - 2kg\cdot\cos\varphi$$

$$\Rightarrow d = \sqrt{(25cm)^2 + (40cm)^2 - 2\cdot 25cm\cdot 40cm\cdot\cos 91,5°}$$

$$= 47,722cm$$

(10)



I : II = ?

• für A_I: $\quad I = \dfrac{w_\beta \cdot c \cdot \sin(\beta/2)}{2}$

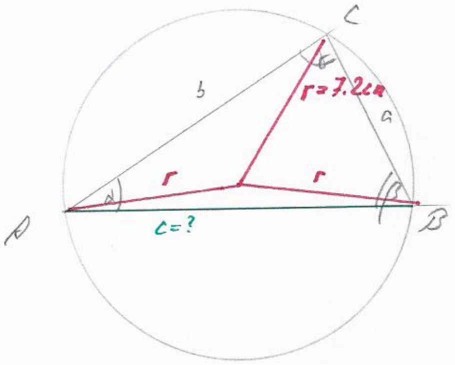
für A_{II}: $\quad II = \dfrac{w_\beta \cdot a \cdot \sin(\beta/2)}{2}$

$\Rightarrow \underline{I : II = c : a}$

• $\dfrac{a}{\sin\alpha} = \dfrac{c}{\sin\gamma} \iff \underline{\dfrac{c}{a} = \dfrac{\sin\gamma}{\sin\alpha}}$

$\Rightarrow$ Teilung $\underline{\underline{I : II = \sin\gamma : \sin\alpha}}$

(11)



$\left.\begin{array}{l} \alpha : \beta : \gamma = 3:4:5 \\ \alpha + \beta + \gamma = 180° \end{array}\right\} \Rightarrow \underline{\alpha = 45°,\ \beta = 60°,\ \gamma = 75°}$

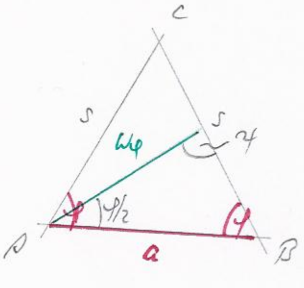
• mit Aufg. 8 folgt: $2r = \dfrac{c}{\sin\gamma}$

$\iff \underline{\underline{c}} = 2r \cdot \sin\gamma = \underline{\underline{13{,}905\,cm}}$

• $\dfrac{a}{\sin\alpha} = \dfrac{c}{\sin\gamma} \Rightarrow \underline{a} = \dfrac{c \cdot \sin\alpha}{\sin\gamma} = \underline{10{,}182\,cm}$

$\Rightarrow \underline{\underline{A_\Delta}} = \dfrac{a \cdot c \cdot \sin\beta}{2} = \underline{\underline{61{,}327\,cm^2}}$

(12)



• $\dfrac{w_\varphi}{\sin\varphi} = \dfrac{a}{\sin\psi} \iff w_\varphi = \dfrac{a \sin\varphi}{\sin\psi}$

• $\psi = 180° - \varphi - \frac{\varphi}{2} = 180° - \frac{3}{2}\cdot\varphi$

$\Rightarrow \underline{\underline{w_\varphi}} = a \cdot \dfrac{\sin\varphi}{\sin(180° - \frac{3}{2}\varphi)} = \underline{\underline{a \cdot \dfrac{\sin\varphi}{\sin(\frac{3}{2}\cdot\varphi)}}}$