Geometrie-Aufgaben: Trigonometrie 6

(1) a) $\dfrac{\sin 2\alpha}{\sin\alpha} = \dfrac{2\cdot\sin\alpha\cdot\cos\alpha}{\sin\alpha} = \underline{\underline{2\cdot\cos\alpha}}$

b) $\dfrac{\sin\alpha+\sin 2\alpha}{1+\cos\alpha+\cos 2\alpha} = \dfrac{\sin\alpha + 2\cdot\sin\alpha\cdot\cos\alpha}{1+\cos\alpha+2\cdot\cos^2\alpha-1}$

$= \dfrac{\sin\alpha\,(1+2\cdot\cos\alpha)}{\cos\alpha\,(1+2\cdot\cos\alpha)} = \underline{\underline{\tan\alpha}}$

c) $\dfrac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\cos(\alpha+\beta)+\cos(\alpha-\beta)} = \dfrac{(\sin\alpha\cdot\cos\beta+\cancel{\sin\beta\cdot\cos\alpha})+(\sin\alpha\cdot\cos\beta-\cancel{\sin\beta\cdot\cos\alpha})}{(\cos\alpha\cdot\cos\beta-\cancel{\sin\alpha\cdot\sin\beta})+(\cos\alpha\cdot\cos\beta+\cancel{\sin\alpha\cdot\sin\beta})}$

$= \dfrac{2\cdot\sin\alpha\cdot\cancel{\cos\beta}}{2\cdot\cos\alpha\cdot\cancel{\cos\beta}} = \underline{\underline{\tan\alpha}}$

(2) a) $\underline{\sin\left(\frac{\pi}{4}+\alpha\right) \overset{!}{=} \cos\left(\frac{\pi}{4}-\alpha\right) \overset{!}{=} \frac{\sin\alpha+\cos\alpha}{\sqrt{2}}}$

Beweis: $\sin\left(\frac{\pi}{4}+\alpha\right) = \cos\left(\frac{\pi}{2}-\left(\frac{\pi}{4}+\alpha\right)\right)$

$= \cos\left(\frac{\pi}{4}-\alpha\right)$ □

$= \cos\frac{\pi}{4}\cdot\cos\alpha + \sin\frac{\pi}{4}\cdot\sin\alpha$

$= \frac{\sqrt{2}}{2}\cdot\cos\alpha + \frac{\sqrt{2}}{2}\cdot\sin\alpha = \frac{1}{\sqrt{2}}\cdot(\cos\alpha+\sin\alpha)$ □

b) $\underline{\tan\left(\frac{\pi}{4}+\alpha\right) \overset{!}{=} \frac{1+\tan\alpha}{1-\tan\alpha}}$

Beweis: $\tan\left(\frac{\pi}{4}+\alpha\right) = \dfrac{\tan\frac{\pi}{4}+\tan\alpha}{1-\tan\frac{\pi}{4}\cdot\tan\alpha} = \dfrac{1+\tan\alpha}{1-\tan\alpha}$ □

c) $\underline{\alpha+\beta+\gamma=180^\circ \overset{!}{\Rightarrow} \tan\alpha+\tan\beta+\tan\gamma = \tan\alpha\cdot\tan\beta\cdot\tan\gamma}$

Beweis: $\alpha+\beta+\gamma=180^\circ \Rightarrow \tan\gamma = \tan(180^\circ-(\alpha+\beta))$

$= -\tan(\alpha+\beta)$

$= -\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\cdot\tan\beta}$

$\Leftrightarrow \tan\gamma\,(1-\tan\alpha\cdot\tan\beta) = -\tan\alpha-\tan\beta$ □

③ a) $\underline{\cos\frac{\pi}{12}} = \cos\left(\frac{\pi}{4}-\frac{\pi}{6}\right)$

$= \cos\frac{\pi}{4}\cdot\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\cdot\sin\frac{\pi}{6}$

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \underline{\underline{\frac{1}{4}\cdot\left(\sqrt{6}+\sqrt{2}\right)}}$

b) $\underline{\sin\frac{5\pi}{12}} = \ldots = \underline{\underline{1/4\cdot\left(\sqrt{6}+\sqrt{2}\right)}}$