

Geometrie-Aufgaben: Trigonometrie 6

① a) $\frac{\sin 2d}{\sin d} = \frac{2 \sin d \cdot \cos d}{\sin d} = \underline{\underline{2 \cdot \cos d}}$

b) $\frac{\sin d + \sin 2d}{1 + \cos d + \cos 2d} = \frac{\sin d + 2 \sin d \cdot \cos d}{1 + \cos d + 2 \cos^2 d - 1}$
 $= \frac{\sin d (1 + 2 \cos d)}{\cos d (1 + 2 \cos^2 d)} = \underline{\underline{\tan d}}$

c) $\frac{\sin(d+\beta) + \sin(d-\beta)}{\cos(d+\beta) + \cos(d-\beta)} = \frac{(\sin d \cdot \cos \beta + \sin \beta \cdot \cos d) + (\sin d \cdot \cos \beta - \sin \beta \cdot \cos d)}{(\cos d \cdot \cos \beta - \sin d \cdot \sin \beta) + (\cos d \cdot \cos \beta + \sin d \cdot \sin \beta)}$
 $= \frac{2 \sin d \cdot \cos \beta}{2 \cos d \cdot \cos \beta} = \underline{\underline{\tan d}}$

② a) $\sin\left(\frac{\pi}{4} + d\right) \stackrel{!}{=} \cos\left(\frac{\pi}{4} - d\right) \stackrel{!}{=} \frac{\cos d + \sin d}{\sqrt{2}}$

Beweis: $\sin\left(\frac{\pi}{4} + d\right) = \cos\left(\frac{\pi}{2} - \left(\frac{\pi}{4} + d\right)\right)$
 $= \cos\left(\frac{\pi}{4} - d\right)$
 $= \cos \frac{\pi}{4} \cdot \cos d + \sin \frac{\pi}{4} \cdot \sin d$
 $= \frac{\sqrt{2}}{2} \cdot \cos d + \frac{\sqrt{2}}{2} \cdot \sin d = \frac{1}{\sqrt{2}} \cdot (\cos d + \sin d)$

b) $\frac{\tan\left(\frac{\pi}{4} + d\right)}{1 - \tan d} \stackrel{!}{=} \frac{1 + \tan d}{1 - \tan d}$

Beweis: $\frac{\tan\left(\frac{\pi}{4} + d\right)}{1 - \tan \frac{\pi}{4} \cdot \tan d} = \frac{\tan \frac{\pi}{4} + \tan d}{1 - \tan \frac{\pi}{4} \cdot \tan d} = \frac{1 + \tan d}{1 - \tan d}$

c) $\underline{\underline{d + \beta + \gamma = 180^\circ}} \stackrel{!}{\Rightarrow} \underline{\underline{\tan d + \tan \beta + \tan \gamma = \tan d \cdot \tan \beta \cdot \tan \gamma}}$

Beweis: $d + \beta + \gamma = 180^\circ \Rightarrow \tan \gamma = \tan(180^\circ - (d + \beta))$
 $= -\tan(d + \beta)$
 $= -\frac{\tan d + \tan \beta}{1 - \tan d \cdot \tan \beta}$

$\Leftrightarrow \tan \gamma (1 - \tan d \cdot \tan \beta) = -\tan d - \tan \beta$

$$\textcircled{3} \text{ a) } \underline{\cos \frac{\pi}{12}} = \cos \left(\frac{\pi}{4} - \frac{\pi}{6} \right)$$

$$= \cos \frac{\pi}{4} \cdot \cos \frac{\pi}{6} + \sin \frac{\pi}{4} \cdot \sin \frac{\pi}{6}$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \underline{\underline{\frac{1}{4} \cdot (\sqrt{6} + \sqrt{2})}}$$

$$\text{b) } \underline{\sin \frac{\pi}{12}} = \dots = \underline{\underline{\frac{1}{4} \cdot (\sqrt{6} - \sqrt{2})}}$$