Geometrie-Aufgaben: Trigonometrie 7

(1) a) $\sin(-\varphi) = \frac{\sqrt{2}}{2} \iff -\varphi = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \pi/4 \vee \frac{3\pi}{4}$

$\iff \underline{\underline{\varphi = -\pi/4 \vee -3\pi/4}}$

b) $\cos(\pi-\varphi) = 1/2 \iff \pi-\varphi = \cos^{-1}(1/2) = \pi/3 \vee -\pi/3$

$\iff \underline{\underline{\varphi = 2\pi/3 \vee 4\pi/3}}$

c) $6\cdot\sin^2\varphi + \sin\varphi = 4\cdot\sin\varphi \iff 3\sin\varphi(2\cdot\sin\varphi - 1) = 0$

$\iff \sin\varphi = 0 \vee 2\cdot\sin\varphi - 1 = 0$

$\iff \underline{\underline{\varphi_{1,2} = 0, \pi}} \vee \sin\varphi = 1/2$

$\iff \underline{\underline{\varphi_{3,4} = \pi/6, 5\pi/6}}$

d) $2\cdot\sin^2\varphi = 3 + 3\cdot\cos\varphi \iff 2\cdot(1-\cos^2\varphi) = 3 + 3\cdot\cos\varphi$

$\overset{x:=\cos\varphi}{\iff} \underline{2x^2 + 3x + 1 = 0}$

$\iff x_{1,2} = \frac{-3 \pm \sqrt{3^2 - 4\cdot 2\cdot 1}}{2\cdot 2} = \frac{-3\pm\sqrt{1}}{4}$

$\Rightarrow x_1 = -1/2 = \cos\varphi \Rightarrow \underline{\varphi_{1,2} = 4\pi/6, 8\pi/6} = \underline{\underline{2\pi/3, 4\pi/3}}$

$x_2 = -1 = \cos\varphi \Rightarrow \underline{\underline{\varphi_3 = \pi}}$

e) $\sin\varphi\cdot\cos\varphi = 0 \iff \sin\varphi = 0 \vee \cos\varphi = 0$

$\iff \underline{\underline{\varphi_{1,2} = 0, \pi \vee \varphi_{3,4} = \pi/2, 3\pi/2}}$

f) $2\cdot\cos^2\varphi = -\cos\varphi \iff \cos\varphi\cdot(2\cos\varphi + 1) = 0$

$\iff \cos\varphi = 0 \vee 2\cos\varphi + 1 = 0$

$\iff \underline{\underline{\varphi_{1,2} = \pi/2, 3\pi/2 \vee \varphi_{3,4} = 2\pi/3, 4\pi/3}}$

g) $\sin^2\varphi - \cos^2\varphi = 0.2 \iff (1-\cos^2\varphi) - \cos^2\varphi = 0.2$

$\iff 2\cos^2\varphi = 0.8$

$\iff \underline{\cos^2\varphi = 0.4}$

$\iff \cos\varphi = +\sqrt{0.4} \vee \cos\varphi = -\sqrt{0.4}$

$\overset{TR}{\iff} \underline{\underline{\varphi_{1,2} = 0.886,\ 5.397}} \vee \underline{\underline{\varphi_{3,4} = 2.256,\ 4.028}}$

h) $\sin^2\varphi + \cos^2\varphi = -0.2$ $\underline{\underline{\lightning}}$

i) $\sin x + \cos x = 0.2 \overset{||^2}{\Rightarrow} \sin^2 x + 2\cdot\sin x\cdot\cos x + \cos^2 x = 0.04 \ldots$

$\iff \sin x = 0.2 - \cos x$

$\overset{||^2}{\Rightarrow} \sin^2 x = 0.04 - 0.4\cdot\cos x + \cos^2 x$

$\iff 1-\cos^2 x = 0.04 - 0.4\cos x + \cos^2 x$

$\overset{q:=\cos x}{\iff} \underline{2q^2 - 0.4q - 0.96 = 0} \Rightarrow q_{1,2} = \dfrac{-(-0.4) \pm \sqrt{(-0.4)^2 - 4\cdot 2\cdot(-0.96)}}{2\cdot 2}$

$= \dfrac{0.4 \pm \sqrt{7.84}}{4}$

$\Rightarrow q_1 = 0.8 = \cos x \Rightarrow \underline{x_{1,2} = 0.644}\ \lightning,\ \underline{\underline{5.640}}$

$q_2 = -0.6 = \cos x \Rightarrow \underline{x_{3,4} = \underline{\underline{2.214}}},\ \underline{4.069}\ \lightning$

j) $6\sin 2x - 3\cdot\tan x = 5\sin x \overset{||:\sin x}{\Rightarrow} 6\cdot 2\cdot\cos x - \dfrac{3}{\cos x} = 5$ $\longrightarrow$ mögl. Verlustumformung $\sin x = 0 \iff x_{1,2} = 0, \pi$

$\overset{||\cdot\cos x =: q}{\Rightarrow} \underline{12q^2 - 5q - 3 = 0} \Rightarrow q_{1,2} = \dfrac{5 \pm \sqrt{(-5)^2 - 4\cdot 12\cdot(-3)}}{2\cdot 12} = \dfrac{5 \pm \sqrt{169}}{24}$

$\Rightarrow q_1 = 3/4 = \cos x \Rightarrow \underline{x_1 = 0.723},\ \underline{x_2 = 5.560}$

$q_2 = -1/3 = \cos x \Rightarrow \underline{x_3 = 1.911},\ \underline{x_4 = 4.376},\ \underline{\underline{x_5 = 0}}$

k) $\tan x + \tan 2x = 0 \quad \Rightarrow$ triviale Lösung $x_1 = 0$

$$\Leftrightarrow \tan x + \frac{2\cdot \tan x}{1-\tan^2 x} = 0 \qquad D = \mathbb{R} \setminus \{ \pi/4, 3\pi/4, 5\pi/4, 7\pi/4 \}$$

$$\overset{D}{\Leftrightarrow} \tan x - \tan^3 x + 2\tan x = 0$$

$$\Leftrightarrow \tan x (3 - \tan^2 x) = 0$$

$$\Leftrightarrow \tan x = 0 \quad \vee \quad \tan^2 x = 1/3 \Leftrightarrow \tan x = \pm\sqrt{1/3} = \pm \frac{\sqrt{3}}{3}$$

$$\Leftrightarrow x_1 = 0,\ x_2 = \pi \quad \vee \quad x_3 = \pi/3,\ x_4 = 2\pi/3,\ x_5 = 4\pi/3,\ x_6 = 5\pi/3$$

e) $\dfrac{\tan 2x}{\tan x} - \dfrac{\tan x}{\tan 2x} = 2 \quad \overset{q := \frac{\tan 2x}{\tan x}}{\Leftrightarrow} \quad q - \dfrac{1}{q} = 2$

$$\Leftrightarrow q^2 - 2q - 1 = 0 \quad \Rightarrow q_{1,2} = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$$

$$\Rightarrow q_1 = 1+\sqrt{2} = \frac{\tan 2x}{\tan x} = \frac{2\cdot \tan x}{(1-\tan^2 x)\cdot \tan x} = \frac{2}{1-\tan^2 x}$$

$$\Leftrightarrow (1+\sqrt{2}) - \tan^2 x (1+\sqrt{2}) = 2$$

$$\Leftrightarrow \tan^2 x = \frac{\sqrt{2}-1}{1+\sqrt{2}} \quad \Rightarrow x_1 = \tan^{-1}\left(\left(\frac{\sqrt{2}-1}{1+\sqrt{2}}\right)^{1/2}\right) = \pi/8$$

$$x_2 = \tan^{-1}\left(-\left(\frac{\sqrt{2}-1}{1+\sqrt{2}}\right)^{1/2}\right) = -\pi/8$$

$$\Rightarrow q_2 = 1-\sqrt{2} = \ldots = \frac{2}{1-\tan^2 x}$$

$$\Leftrightarrow \tan^2 x = \frac{-\sqrt{2}-1}{1-\sqrt{2}} = \frac{1+\sqrt{2}}{\sqrt{2}-1} \quad \Rightarrow x_3 = \tan^{-1}\left(\left(\frac{1+\sqrt{2}}{\sqrt{2}-1}\right)^{1/2}\right) = \frac{3\pi}{8}$$

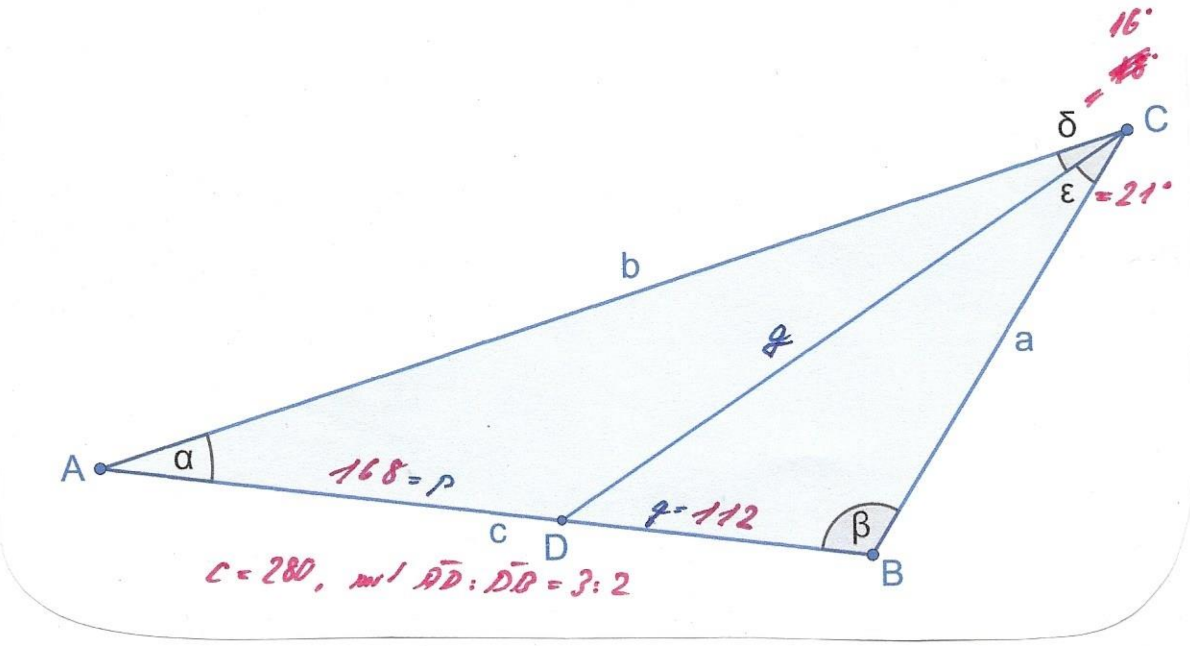
$$x_4 = \tan^{-1}\left(-\left(\frac{1+\sqrt{2}}{\sqrt{2}-1}\right)^{1/2}\right) = -\frac{3\pi}{8}$$

$$\Rightarrow \text{Lög.:} \quad x_1 = \pi/8 \Rightarrow \hat{x}_1 = \frac{9\pi}{8}, \quad x_3 = 3\pi/8 \Rightarrow \hat{x}_3 = \frac{11\pi}{8}$$

$$x_2 = -\pi/8 \mathrel{\hat{=}} 15\pi/8 \Rightarrow \hat{x}_2 = \frac{23\pi}{8} \mathrel{\hat{=}} \frac{7\pi}{8}$$

$$x_4 = -3\pi/8 \mathrel{\hat{=}} 13\pi/8 \Rightarrow \hat{x}_4 = \frac{21\pi}{8} \mathrel{\hat{=}} \frac{5\pi}{8}$$

2.



$\underline{\delta + \varepsilon = 37°}$

- $\frac{p}{\sin\delta} = \frac{g}{\sin\alpha} \iff \underline{g = \frac{p\cdot\sin\alpha}{\sin\delta}}$

- $\frac{q}{\sin\varepsilon} = \frac{g}{\sin\beta} \iff \underline{g} = \frac{q\cdot\sin\beta}{\sin\varepsilon} = \frac{q\cdot\sin(180° - (\alpha+\delta+\varepsilon))}{\sin\varepsilon} = \frac{q\cdot\sin(\alpha+(\delta+\varepsilon))}{\sin\varepsilon)} =$

$$= \underline{\frac{q\cdot(\sin\alpha\cdot\cos(\delta+\varepsilon) + \sin(\delta+\varepsilon)\cdot\cos\alpha)}{\sin\varepsilon}}$$

$$\Rightarrow \frac{p\cdot\sin\alpha}{\sin\delta} = \frac{q\cdot\sin\alpha\cdot\cos(\delta+\varepsilon)}{\sin\varepsilon} + \frac{q\cdot\sin(\delta+\varepsilon)\cdot\cos\alpha}{\sin\varepsilon}$$

$$\iff \sin\alpha\left(\frac{p}{\sin\delta} - \frac{q\cdot\cos(\delta+\varepsilon)}{\sin\varepsilon}\right) = \cos\alpha\cdot\frac{q\cdot\sin(\delta+\varepsilon)}{\sin\varepsilon} \quad // : \cos\alpha$$

$$\tan\alpha = \frac{\frac{q\cdot\sin(\delta+\varepsilon)}{\sin\varepsilon}}{\frac{p}{\sin\delta} - \frac{q\cdot\cos(\delta+\varepsilon)}{\sin\varepsilon}}$$

$$\Rightarrow \underline{\underline{\alpha = 27{,}592°}}$$

$$\Rightarrow \underline{\underline{\beta = 115{,}4°,\ a = 216,\ b = 420}}$$

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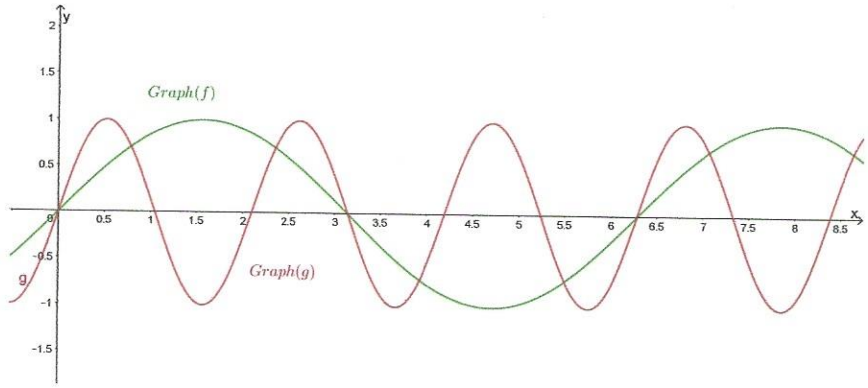

$\Rightarrow \quad f(x) = \sin x \; , \quad g(x) = \sin 3x$

$$\Rightarrow \quad f(x) = g(x) \iff \sin x = \sin 3x = \sin(x + 2x)$$

$$= \sin x \cdot \cos 2x + \sin 2x \cdot \cos x$$

$$= \sin x \cdot (2 \cdot \cos^2 x - 1) + 2 \cdot \sin x \cdot \cos x \cdot \cos x$$

Verlustumformung:
$\sin x = 0$
$\Rightarrow \underline{\underline{x_1 = 0 \; , \; x_2 = \pi}}$

$$\iff 1 = 2 \cdot \cos^2 x - 1 + 2 \cdot \cos^2 x$$

$$\iff 2 = 4 \cdot \cos^2 x$$

$$\Rightarrow \cos x = \pm\sqrt{1/2} \quad \Rightarrow \quad \underline{\underline{x_3}} = \cos^{-1}\left(\sqrt{1/2}\right) = \underline{\underline{0{,}785}}$$

$$\underline{\underline{x_4}} = \cos^{-1}\left(-\sqrt{1/2}\right) = \underline{\underline{2{,}356}}$$

$\Rightarrow$ E'kreis liefert lösg $> \pi$