Vektorgeometrie:

(Aufgaben: Singerist & Wirth)

① $\vec{g}(t) = \begin{pmatrix}1\\1\\4\end{pmatrix} + t\cdot\begin{pmatrix}4\\1\\1\end{pmatrix}$, $\vec{h}(s) = \begin{pmatrix}5\\5\\2\end{pmatrix} + s\cdot\begin{pmatrix}0\\1\\-1\end{pmatrix}$

a) Schnittpunkt $\Rightarrow \vec{g}(t) = \vec{h}(s)$... & Schnittwinkel $\Rightarrow$ SP Richtungsvektoren...

· $\vec{g}(t) = \vec{h}(s) \iff \begin{pmatrix}1\\1\\4\end{pmatrix} + t\cdot\begin{pmatrix}4\\1\\1\end{pmatrix} = \begin{pmatrix}5\\5\\2\end{pmatrix} + s\cdot\begin{pmatrix}0\\1\\-1\end{pmatrix}$

$$\iff \begin{cases} 1 + 4t = 5 & \Rightarrow t = 1 \\ 1 + t = 5 + s & \Rightarrow s = -3 \\ 4 + t = 2 - s \end{cases}$$

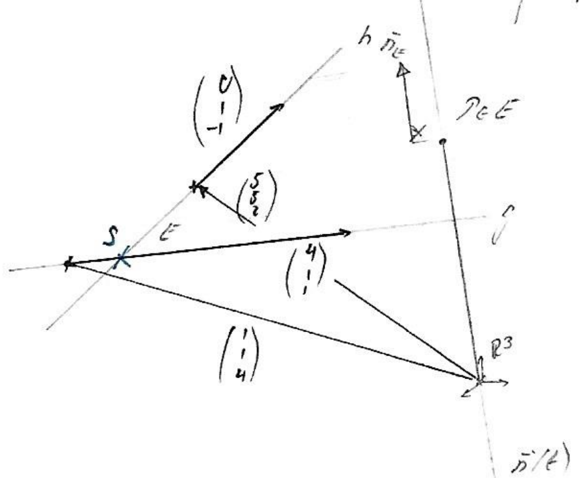
überprüfen: $4 + 1 = 2 - (-3)$ passt!

$\Rightarrow \vec{OS} = \vec{g}(1) = \vec{h}(-3) = \begin{pmatrix}5\\2\\5\end{pmatrix}$

$\Rightarrow \underline{S = (5|2|5)}$

· $\begin{pmatrix}4\\1\\1\end{pmatrix}\cdot\begin{pmatrix}0\\1\\-1\end{pmatrix} = \left|\begin{pmatrix}4\\1\\1\end{pmatrix}\right|\cdot\left|\begin{pmatrix}0\\1\\-1\end{pmatrix}\right|\cdot\cos\varphi \Rightarrow \varphi = \cos^{-1}\left(\frac{4\cdot 0 + 1\cdot 1 + 1\cdot(-1)}{\sqrt{18}\cdot\sqrt{2}}\right) = \underline{90^\circ}$

3) $d(0,E) \Rightarrow$



$\Rightarrow$ · $\vec{E}(r,s) = \vec{OS} + r\cdot\begin{pmatrix}4\\1\\1\end{pmatrix} + s\cdot\begin{pmatrix}0\\1\\-1\end{pmatrix}$

· $\vec{n}(t) = \begin{pmatrix}0\\0\\0\end{pmatrix} + t\cdot\vec{n}_E$

· $\vec{n}\cap E \Rightarrow P$

$\Rightarrow \underline{d(0,E) = d(0,P)}$

· $\vec{E}(r,s) = \begin{pmatrix}5\\2\\5\end{pmatrix} + r\cdot\begin{pmatrix}4\\1\\1\end{pmatrix} + s\cdot\begin{pmatrix}0\\1\\-1\end{pmatrix}$

$\Rightarrow \underline{\vec{n}_E} = \begin{pmatrix}4\\1\\1\end{pmatrix}\times\begin{pmatrix}0\\1\\-1\end{pmatrix} = \begin{pmatrix}-1-1\\0-(-4)\\4-0\end{pmatrix} = \underline{\begin{pmatrix}-2\\4\\4\end{pmatrix}}$ $\Rightarrow E: -2x + 4y + 4z - d = 0$

$\begin{matrix}4 & 0\\1 & 1\\1 & -1\end{matrix}$

$S\in E \Rightarrow (-2)\cdot 5 + 4\cdot 2 + 4\cdot 5 - d = 0$

$\Rightarrow \underline{E: -2x + 4y + 4z - 18 = 0}$

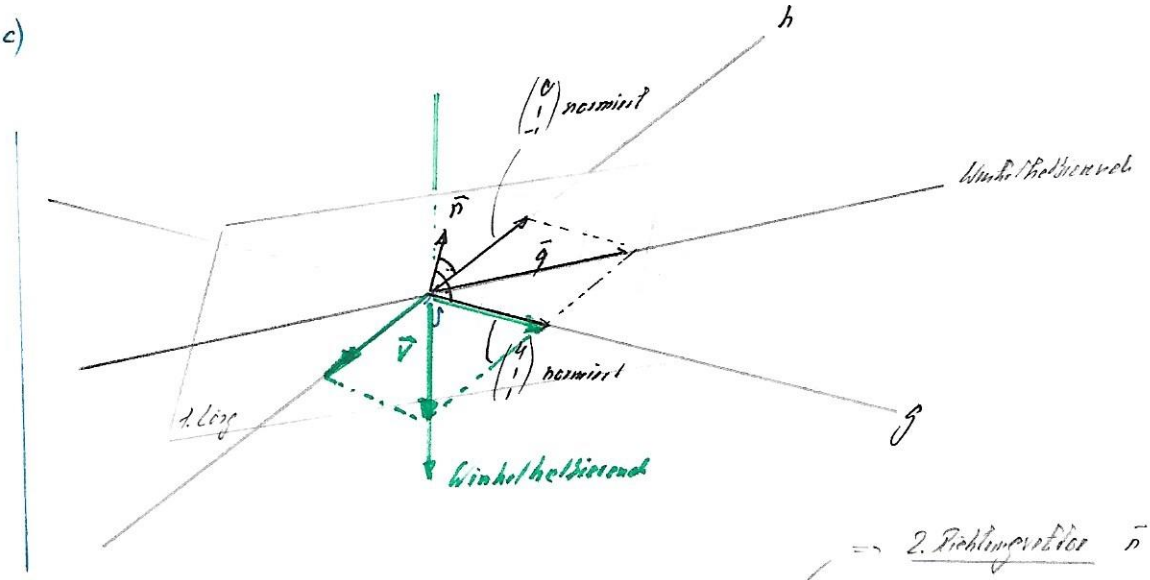
· $\underline{\vec{n}(t) = \begin{pmatrix}0\\0\\0\end{pmatrix} + t\cdot\begin{pmatrix}-2\\4\\4\end{pmatrix}}$

· $\vec{n}\cap E \Rightarrow (-2)\cdot(-2t) + 4\cdot 4t + 4\cdot 4t - 18 = 0$

$\iff \underline{t = 1/2} \Rightarrow \vec{OP} = \vec{n}(1/2) = \begin{pmatrix}-1\\2\\2\end{pmatrix} \Rightarrow \underline{P = (-1|2|2)}$

· $\underline{d(0,E)} = d(0,P) = \sqrt{(-1)^2 + 2^2 + 2^2} = \underline{3}$

c)



$\Rightarrow$ 2. Richtungsvektor $\vec{n} = \begin{pmatrix}4\\1\\1\end{pmatrix} \times \begin{pmatrix}0\\1\\-1\end{pmatrix} = \begin{pmatrix}-2\\4\\4\end{pmatrix}$

1. Lsg: Ebene durch die Winkelhalbierende & $\perp$ zu E

$\Downarrow$

1. Richtungsvektor $\vec{q} = \frac{1}{\sqrt{2}} \cdot \begin{pmatrix}0\\1\\-1\end{pmatrix} + \frac{1}{\sqrt{18}} \begin{pmatrix}4\\1\\1\end{pmatrix} = \begin{pmatrix}0.943\\0.943\\-0.471\end{pmatrix} \Rightarrow \begin{pmatrix}2\\2\\-1\end{pmatrix}$

$\Rightarrow \vec{E_1}(r,s) = \vec{OS} + r \cdot \vec{q} + s \cdot \vec{n} = \begin{pmatrix}5\\2\\5\end{pmatrix} + r \begin{pmatrix}2\\2\\-1\end{pmatrix} + s \cdot \begin{pmatrix}-2\\4\\4\end{pmatrix}$

$\vec{n_{E_1}} = \begin{pmatrix}2\\2\\-1\end{pmatrix} \times \begin{pmatrix}-2\\4\\4\end{pmatrix} = \begin{pmatrix}12\\-6\\12\end{pmatrix} \Rightarrow E_1:\ 2x - y + 2z - 18 = 0$

2. Lsg. (mit 2. Winkelhalbierende)

$\Rightarrow$ 1. Richtungsvektor $\vec{v} = -\frac{1}{\sqrt{2}} \begin{pmatrix}0\\1\\-1\end{pmatrix} + \frac{1}{\sqrt{18}} \begin{pmatrix}4\\1\\1\end{pmatrix} = \begin{pmatrix}0.943\\-0.471\\0.943\end{pmatrix} \Rightarrow \begin{pmatrix}2\\-1\\2\end{pmatrix}$

2. Richtungsvektor $\vec{n} = \begin{pmatrix}-2\\4\\4\end{pmatrix}$

$\Rightarrow \vec{E_2}(r,s) = \vec{OS} + r \cdot \vec{v} + s \cdot \vec{n} = \begin{pmatrix}5\\2\\5\end{pmatrix} + r \cdot \begin{pmatrix}2\\-1\\2\end{pmatrix} + s \cdot \begin{pmatrix}-2\\4\\4\end{pmatrix}$

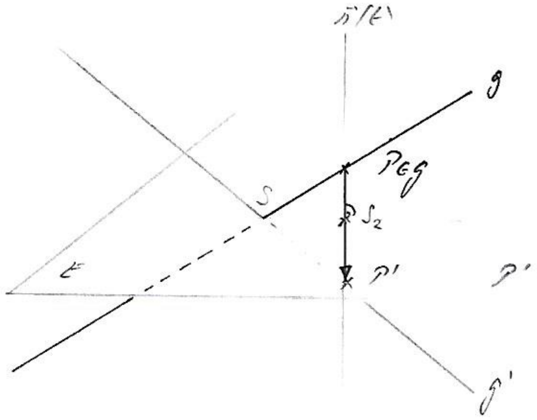
$\vec{n_{E_2}} = \begin{pmatrix}2\\-1\\2\end{pmatrix} \times \begin{pmatrix}-2\\4\\4\end{pmatrix} = \begin{pmatrix}-12\\-12\\6\end{pmatrix} \Rightarrow E_2:\ 2x + 2y - z - 9 = 0$

(3.) $E: 2x+2y+z-30=0 \quad , \quad \vec{g}(t) = \begin{pmatrix}3\\6\\3\end{pmatrix} + t\cdot\begin{pmatrix}1\\0\\1\end{pmatrix}$

a) Neigungswinkel $\Rightarrow \varphi = \sin^{-1}\left(\frac{|\vec{n}_E\cdot\vec{a}|}{|\vec{n}_E|\cdot|\vec{a}|}\right)$

$$\left.\begin{array}{l} \vec{n}_E = \begin{pmatrix}2\\2\\1\end{pmatrix} \Rightarrow |\vec{n}_E| = \sqrt{9} \\ \vec{a} = \begin{pmatrix}1\\0\\1\end{pmatrix} \Rightarrow |\vec{a}| = \sqrt{2} \end{array}\right\} \Rightarrow \varphi = \sin^{-1}\left(\frac{|2\cdot1+2\cdot0+1\cdot1|}{3\cdot\sqrt{2}}\right) = \underline{\underline{45^\circ}}$$

b) Spiegelungsgerade



P' = P gespiegelt an E.

- $g \cap E \Rightarrow 2\cdot(3+1t)+2\cdot(6+0t)+1\cdot(3+1t)-30=0$
$$\Leftrightarrow \quad 3t = 9$$
$$\Leftrightarrow \quad \underline{t=3} \quad \Rightarrow \quad \vec{OS} = \vec{g}(3) = \underline{\begin{pmatrix}6\\6\\6\end{pmatrix}} \quad , \quad \underline{S=(6|6|6)}$$

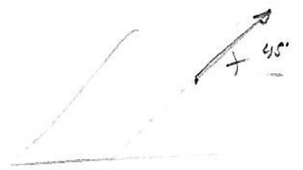
- $P \in g$ beliebig $\Rightarrow P=(3|6|3)$
$$\Rightarrow \underline{\vec{n}(t) = \begin{pmatrix}3\\6\\3\end{pmatrix} + t\cdot\begin{pmatrix}2\\2\\1\end{pmatrix}}$$

$n \cap E \Rightarrow 2\cdot(3+2t)+2\cdot(6+2t)+1\cdot(3+t)-30=0$
$$\Leftrightarrow \quad 9t = 9$$
$$\Leftrightarrow \quad \underline{t=1} \quad \Rightarrow \quad \vec{OS_2} = \vec{n}(1)$$
$$\Rightarrow \quad \underline{\vec{OP'} = \vec{n}(2\cdot1)} = \underline{\begin{pmatrix}7\\10\\5\end{pmatrix}} \quad , \quad \underline{P'=(7|10|5)}$$

$$\Rightarrow \underline{\vec{g'}(t)} = \vec{OS} + t\cdot\vec{SP'}$$
$$\underline{\underline{= \begin{pmatrix}6\\6\\6\end{pmatrix} + t\begin{pmatrix}1\\4\\-1\end{pmatrix}}}$$

c) Lösen: · Idee g schneiden mit Ebenen, die im Abstand von 6 ‖ zu E verlaufen

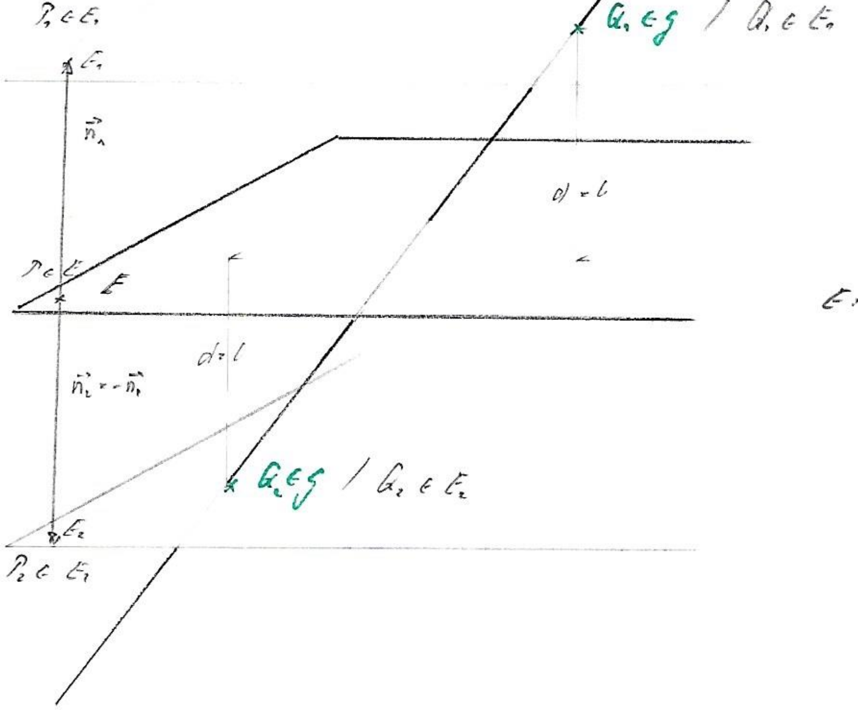
· Spez. Situation



$\Rightarrow$ Erzeuge Richtungsvektor der Länge $\sqrt{2}\cdot6$

d) mit Zylinder

c) Punkte auf g mit Abstand 6 zu E



$E: 2x + 2y + z - 30 = 0$

· $\underline{P \in E}$ ⇒ z.B. $\underline{P = (1/2/24)}$

$2\cdot 1 + 2\cdot 2 + z = 30$

· $\vec{n}_1 = \lambda \cdot \vec{n}_E$, mit $\lambda = \frac{6}{|\vec{n}_E|}$

$\vec{n}_E = \begin{pmatrix}2\\2\\1\end{pmatrix}$, $|\vec{n}_E| = \sqrt{9} = 3$

$\Rightarrow \underline{\vec{n}_1 = 2\cdot\begin{pmatrix}2\\2\\1\end{pmatrix}} \Rightarrow \vec{OP_1} = \vec{OP} + \vec{n}_1 = \begin{pmatrix}5\\6\\26\end{pmatrix}$

$\Rightarrow E_1 = 2x + 2y + z - d = 0$

$P_1 \in E_1 \Rightarrow 2\cdot 5 + 2\cdot 6 + 26 = d$

$\Rightarrow E_1: \underline{2x + 2y + z - 48 = 0}$

$g \cap E_1 \Rightarrow 2\cdot(3+t) + 2\cdot 6 + 1\cdot(3+t) - 48 = 0$

$\Leftrightarrow 6 + 2t + 12 + 3 + t = 48$

$\Leftrightarrow \underline{t} = \underline{\frac{27}{3} = 9} \Rightarrow \vec{OQ_1} = \vec{g}(9) = \begin{pmatrix}12\\6\\12\end{pmatrix}$

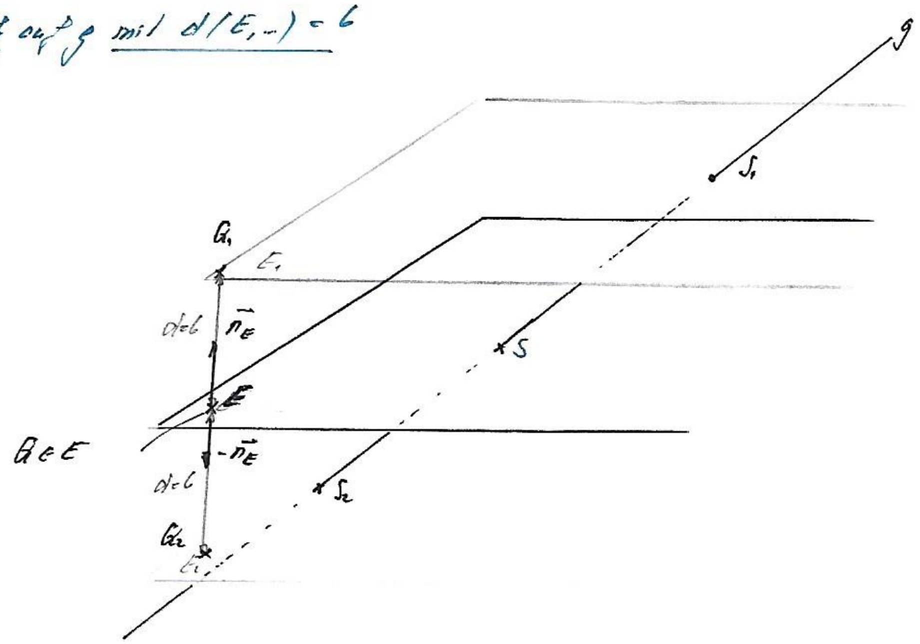
$\Rightarrow \underline{\underline{Q_1 = (12/6/12)}}$

· $\vec{n}_2 = (-2)\cdot\begin{pmatrix}2\\2\\1\end{pmatrix} \Rightarrow P_2 = (-3/-2/22) \Rightarrow E_2: 2x + 2y + z - 12 = 0$

$g \cap E_2 \Rightarrow 2\cdot(3+t) + 2\cdot 6 + 1\cdot(3+t) - 12 = 0 \Rightarrow \underline{t = -3} \Rightarrow \vec{OQ_2} = \vec{g}(-3) \Rightarrow \underline{\underline{Q_2 = (0/6/0)}}$

c) Zwei Punkte auf g mit $d(E, ..) = 6$

parallele Ebenen im Abstand 6 schneiden mit g $\Rightarrow S_1, S_2$



· $G \in E$ bel. $\Rightarrow G = (2|2|22)$

· $\vec{n}_E$ der Länge 6, $\vec{n}_E = \begin{pmatrix}2\\2\\1\end{pmatrix}$, $|\vec{n}_E| = \sqrt{9} = 3$ $\Bigg\} \Rightarrow \vec{n}_6 = 6 \cdot \frac{1}{3} \cdot \begin{pmatrix}2\\2\\1\end{pmatrix} = \begin{pmatrix}4\\4\\2\end{pmatrix}$

· $S_1 \in E_1$: $\vec{OG_1} = \vec{OG} + \vec{n}_6 = \begin{pmatrix}6\\6\\24\end{pmatrix}$

$E_1 \parallel E \Rightarrow E_1: 2x + 2y + z - d = 0$

$G_1 \in E \Rightarrow E_1: 2x + 2y + z - 48 = 0$

$g \cap E_1 \Rightarrow 2 \cdot (3 + t) + 2 \cdot 6 + (3 + t) - 48 = 0$

$\Leftrightarrow 3t = 27$

$\Leftrightarrow t = 9 \Rightarrow \vec{OS_1} = \vec{g}(11) = \begin{pmatrix}12\\6\\12\end{pmatrix}$

$\Rightarrow S_1 = (12|6|12)$

· $S_2 \in E_2$: $\vec{OG_2} = \vec{OG} - \vec{n}_6 = \begin{pmatrix}-2\\-2\\20\end{pmatrix}$

$E_2 \parallel E \Rightarrow E_2: 2x + 2y + z - d = 0$

$G_2 \in E_2 \Rightarrow E_2: 2x + 2y + z - 12 = 0$

$g \cap E_2 \Rightarrow 2(3 + t) + 2 \cdot 6 + (3 + t) - 12 = 0$

$\Leftrightarrow 3t = -9$

$\Leftrightarrow t = -3 \Rightarrow \vec{OS_2} = \vec{g}(-3) = \begin{pmatrix}0\\6\\0\end{pmatrix}$

$\Rightarrow S_2 = (0|6|0)$

(4) 1 Kugelgleichung: $(x-a)^2+(y-b)^2+(z-c)^2=r^2$

für eine Kugel, mit $M=(a|b|c)$, Radius $=r$.

$\iff x^2+y^2+z^2-2ax-2by-2cz+a^2+b^2+c^2=r$

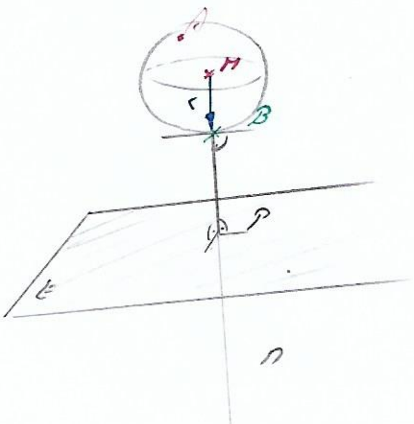
$\iff (\vec{OP}-\vec{OM})^2=r^2$, mit $P \in$ Kugel bekannt.)

Geg.: Kugel, mit M = Mittelpunkt $=(2|4|0)$, $A=(4|5|-2) \in$ Kugel

Ebene $E: x+2y+2z+26=0$

a) $\underline{r} = d(M,A) = \sqrt{(4-2)^2+(5-4)^2+(-2)^2} = \underline{\underline{3}}$

b) Abstand "⊥ zur Kugel" ⇒ ges. Punkt liegt auf einer Geraden, ⊥ zu E durch M



i) $n: n(t) = \vec{OM} + t\cdot\vec{n}_E = \begin{pmatrix}2\\4\\0\end{pmatrix} + t\cdot\begin{pmatrix}1\\2\\2\end{pmatrix}$

ii) $n \cap E \iff (2+t)+2\cdot(4+2t)+2\cdot 2t+26=0$

$\iff 2+t+8+4t+4t+26=0$

$\iff 9t = 36$

$\iff t=4$

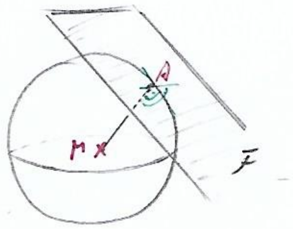
$\Rightarrow \vec{OP} = \vec{n}(4) = \begin{pmatrix}6\\12\\8\end{pmatrix}$

iii) $\underline{\underline{d(B,P)}} = d(M,P) - r$

$= \sqrt{(6-2)^2+(12-4)^2+8^2} - 3$

$= \sqrt{16+64+64} - 3 = \underline{\underline{9}}$

c)



Ebene F ⊥ $\vec{MA} = \begin{pmatrix}2\\1\\-2\end{pmatrix}$

$\Rightarrow F: 2x+y-2z-d=0$

$A \in F \Rightarrow 2\cdot 4+5-2\cdot(-2)-d=0$

$\iff \underline{d=17}$

$\Rightarrow F: 2x+y-2z-17=0$

$\sphericalangle(F,E) := \sphericalangle(\vec{n}_F,\vec{n}_E)$

$= \cos^{-1}\left(\dfrac{\begin{pmatrix}2\\1\\-2\end{pmatrix}\cdot\begin{pmatrix}1\\2\\2\end{pmatrix}}{\sqrt{9}\cdot\sqrt{9}}\right) = \cos^{-1}\left(\dfrac{0}{9}\right) = \underline{\underline{90° = \pi}}$

d) $E \cap F = s$, mit $s: \vec{s}(t) = \vec{OS} + t \cdot (\vec{n}_E \times \vec{n}_F)$

$$\underline{\vec{n}_E \times \vec{n}_F} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} -4-2 \\ 4-(-2) \\ 1-4 \end{pmatrix} = \underline{\begin{pmatrix} -6 \\ 6 \\ -3 \end{pmatrix}}$$

$\Rightarrow$ für Gerade g gilt: $\underline{\vec{g}(t)} = \vec{OM} + t \cdot (\vec{n}_E \times \vec{n}_F)$

$$= \underline{\begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} + t \cdot \begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix}}$$

$g \cap$ Kugel $\Leftrightarrow \left| t \cdot \begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix} \right| = r$

$\Leftrightarrow |t \cdot 3| = 3$

$\Leftrightarrow \underline{t = \pm 1}$

$\Rightarrow$ für die ges. Durchstoßpunkte gilt: $\vec{OP_1} = \vec{g}(-1) \Rightarrow \underline{\underline{P_1 = (4/2/1)}}$

$\vec{OP_2} = \vec{g}(1) \Rightarrow \underline{\underline{P_2 = (0/6/-1)}}$

(mit $\underline{\text{Kugelgleichung}}$: $((2-2t)-2)^2 + ((4+2t)-4)^2 + (-t-0)^2 = 3^2$

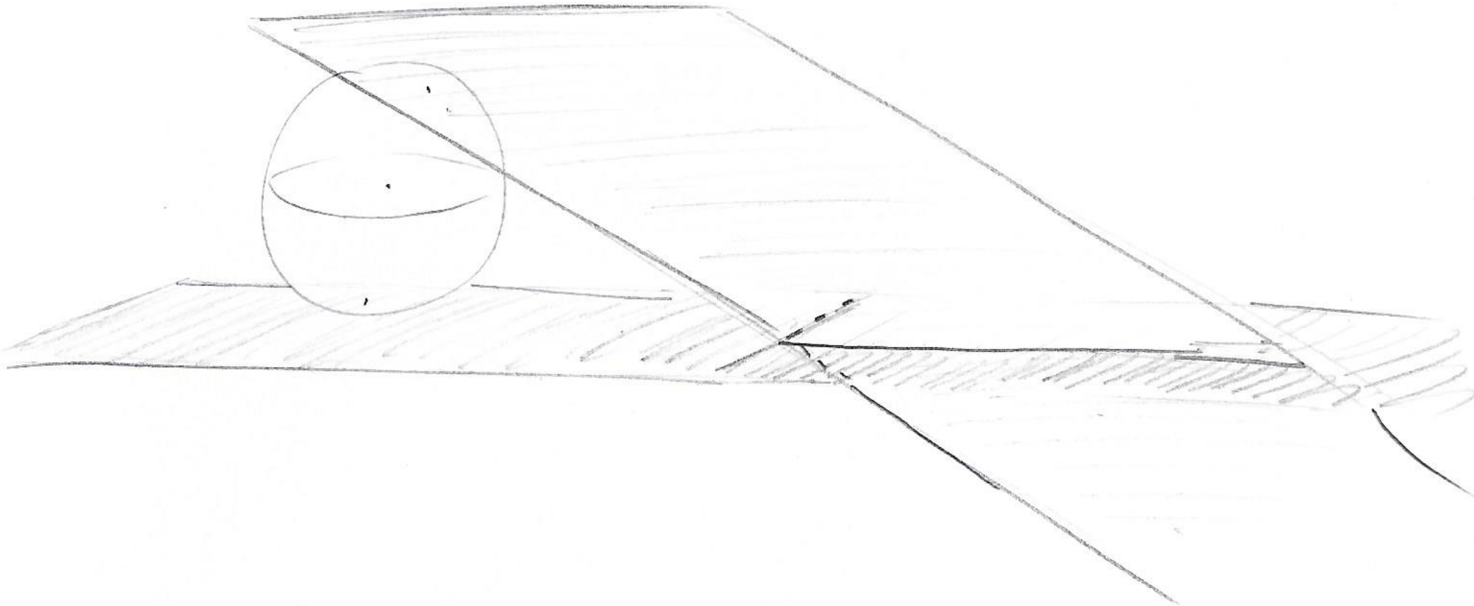
$\Leftrightarrow (-2t)^2 + (2t)^2 + (-t)^2 = 9$

$\Leftrightarrow 9t^2 = 9$

$\Leftrightarrow \underline{t = \pm 1}$

⋮)

4d)



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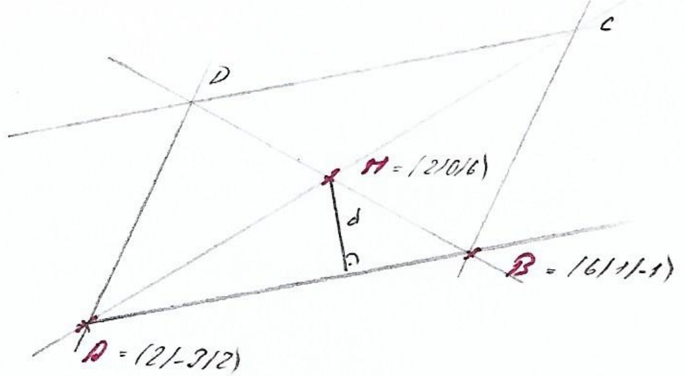


a) die Ecken C & D:

$$\cdot\ \vec{OC} = \vec{OA} + 2\cdot\vec{AM} = \begin{pmatrix}2\\-3\\2\end{pmatrix} + 2\cdot\begin{pmatrix}0\\3\\4\end{pmatrix} = \begin{pmatrix}2\\3\\10\end{pmatrix} \Rightarrow \underline{\underline{C = (2|3|10)}}$$

$$\cdot\ \vec{OD} = \vec{OB} + 2\cdot\vec{BM} = \begin{pmatrix}6\\1\\-1\end{pmatrix} + 2\cdot\begin{pmatrix}-4\\-1\\7\end{pmatrix} = \begin{pmatrix}-2\\-1\\13\end{pmatrix} \Rightarrow \underline{\underline{D = (-2|-1|13)}}$$

b) Schnittwinkel PG ∩ xy-Ebene.

· Normalenvektoren $\cdot\ \vec{n}_{PG} = \vec{AB} \times \vec{AD}$

$$= \begin{pmatrix}4\\4\\-3\end{pmatrix} \times \begin{pmatrix}-4\\2\\11\end{pmatrix} = \begin{pmatrix}44-(-6)\\12-44\\8-(-16)\end{pmatrix} = \begin{pmatrix}50\\-32\\24\end{pmatrix} \Rightarrow \vec{n}_1 = \begin{pmatrix}25\\-16\\12\end{pmatrix},\ |\vec{n}_1| = \sqrt{1025} = 32.016$$

$$\cdot\ \vec{n}_{xy} = \begin{pmatrix}0\\0\\1\end{pmatrix},\quad |\vec{n}_2| = 1$$

$$\Rightarrow \varphi = \cos^{-1}\left(\frac{\vec{n}_1\cdot\vec{n}_2}{n_1\cdot n_2}\right) = \cos^{-1}\left(\frac{12}{\sqrt{32.016}}\right) = \underline{\underline{1.187 = 67.987^\circ}}$$

c) $$\underline{\underline{d}} = \frac{|\vec{AB}\times\vec{AM}|}{|\vec{AB}|} = \frac{\left|\begin{pmatrix}4\\4\\-3\end{pmatrix}\times\begin{pmatrix}0\\3\\4\end{pmatrix}\right|}{\sqrt{4^2+4^2+(-3)^2}} = \frac{\left|\begin{pmatrix}16-(-9)\\0-16\\12-0\end{pmatrix}\right|}{\sqrt{41}} = \frac{\sqrt{25^2+16^2+12^2}}{\sqrt{41}}$$

$$= \frac{\sqrt{1025}}{\sqrt{41}} = \underline{\underline{5}}$$

d) $M \in x\text{-Achse} \Rightarrow M = (x|0|0)$

$ABCD = \text{Rhombus} \Rightarrow \vec{AM} \perp \vec{BM}$

$$\Leftrightarrow \vec{AM}\cdot\vec{BM} = 0$$

$$\Leftrightarrow \begin{pmatrix}x-2\\3\\-2\end{pmatrix}\cdot\begin{pmatrix}x-6\\-1\\1\end{pmatrix} = (x-2)(x-6) + 3\cdot(-1) + (-2)\cdot 1 \overset{!}{=} 0$$

$$\Leftrightarrow x^2 - 8x + 12 - 3 - 2 = 0$$

$$\Leftrightarrow x^2 - 8x + 7 = 0$$

$$\Leftrightarrow \underline{(x-1)(x-7) = 0} \Rightarrow \underline{\underline{M_1 = (1|0|0)}}$$

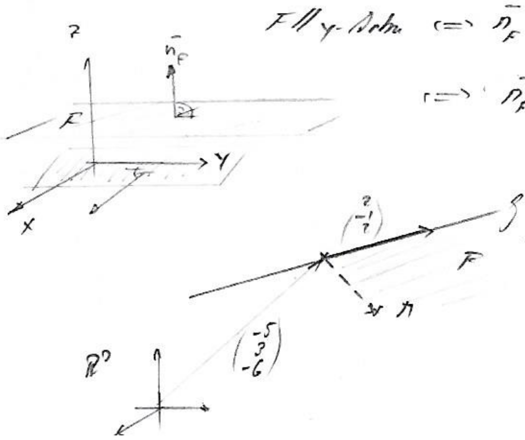
$$\underline{\underline{M_2 = (7|0|0)}}$$

7 $\quad g: \vec{g}(t) = \begin{pmatrix}-5\\3\\-6\end{pmatrix} + t\cdot\begin{pmatrix}2\\-1\\2\end{pmatrix}, \quad M = (2|4|1)$

a) Neigungswinkel g, xy-Ebene:

$$\text{Def.} \Rightarrow \varphi = \sin^{-1}\left(\frac{\left|\begin{pmatrix}2\\-1\\2\end{pmatrix}\cdot\begin{pmatrix}0\\0\\1\end{pmatrix}\right|}{\sqrt{9}\cdot\sqrt{1}}\right) = \sin^{-1}\left(\frac{2}{3}\right) = \underline{\underline{0.730 = 41.810°}}$$

b) Ebene durch g und M ∥ y-Achse:



$$F \parallel y\text{-Achse} \iff \vec{n}_F \perp y\text{-Achse}$$
$$\iff \vec{n}_F\cdot\begin{pmatrix}0\\1\\0\end{pmatrix} \overset{!}{=} 0$$

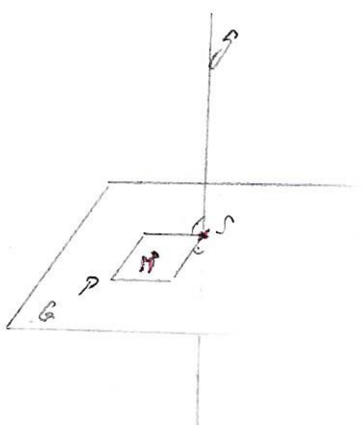
$$F: \vec{F}(r,s) = \begin{pmatrix}-5\\3\\-6\end{pmatrix} + r\begin{pmatrix}2\\-1\\2\end{pmatrix} + s\cdot\begin{pmatrix}2-(-5)\\4-3\\1-(-6)\end{pmatrix}$$
$$= \begin{pmatrix}-5\\3\\-6\end{pmatrix} + r\cdot\begin{pmatrix}2\\-1\\2\end{pmatrix} + s\cdot\begin{pmatrix}7\\1\\7\end{pmatrix}$$

$$\Rightarrow \vec{n}_F = \begin{pmatrix}2\\-1\\2\end{pmatrix}\times\begin{pmatrix}7\\1\\7\end{pmatrix} = \begin{pmatrix}-7-2\\14-14\\2-(-7)\end{pmatrix} = \begin{pmatrix}-9\\0\\9\end{pmatrix}$$

$$\begin{pmatrix}2\\-1\\2\end{pmatrix} \quad \begin{pmatrix}7\\1\\7\end{pmatrix}$$

$$\Rightarrow \vec{n}_F\cdot\begin{pmatrix}0\\1\\0\end{pmatrix} = 0 \ ✓$$

c)



$$\left.\begin{array}{l} \cdot\ \underline{G \perp \text{zu } g \text{ mit } M \in G:} \Rightarrow 2x - y + 2z - d = 0 \\ \qquad \cdot\ 2\cdot 2 - 4 + 2\cdot 1 - d = 0 \end{array}\right\} \Rightarrow$$

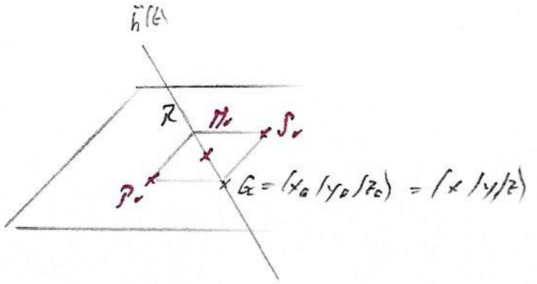
$$\underline{G: 2x - y + 2z - 2 = 0}$$

$$\cdot\ \underline{S \in G \cap g:} \quad 2\cdot(-5+2t) - 1\cdot(3-t) + 2\cdot(-6+2t) - 2 = 0$$
$$\iff -10 + 4t - 3 + t - 12 + 4t - 2 = 0$$
$$\iff 9t = +27$$
$$\iff \underline{t = 3}$$

$$\Rightarrow \underline{\vec{OS} = \vec{g}(3) \Rightarrow \underline{\underline{S = (1|0|0)}}}$$

$$\Rightarrow \vec{OP} = \vec{OM} + \vec{SM}$$
$$= \begin{pmatrix}2\\4\\1\end{pmatrix} + \begin{pmatrix}2-1\\4-0\\1-0\end{pmatrix} = \begin{pmatrix}3\\8\\2\end{pmatrix} \Rightarrow \underline{\underline{P = (3|8|2)}}$$

7 c) Situation



für G: · $\vec{OS} + \vec{SG} = \vec{OP} + \vec{PG}$

$$\Leftrightarrow \begin{pmatrix}1\\0\\0\end{pmatrix} + \begin{pmatrix}x-1\\y-0\\z-0\end{pmatrix} = \begin{pmatrix}3\\8\\2\end{pmatrix} + \begin{pmatrix}x-3\\y-8\\z-2\end{pmatrix}$$

$$\Leftrightarrow \quad x = 3 + x - 3$$

$\vdots$ bringt nichts!

· $\vec{PG} \perp \vec{SG} \Leftrightarrow \begin{pmatrix}x-3\\y-8\\z-2\end{pmatrix} \cdot \begin{pmatrix}x-1\\y-0\\z-0\end{pmatrix} = 0$

$$\Leftrightarrow (x-3)(x-1) + y\cdot(y-8) + z\cdot(z-2) = 0$$

$\vdots$

· $PG = SG \Leftrightarrow (x-3)^2 + (y-8)^2 + (z-2)^2 = (x-1)^2 + y^2 + z^2$

$$\Leftrightarrow (x^2-6x+9) + (y^2-16y+64) + (z^2-4z+4) = (x^2-2x+1) + y^2 + z^2$$

$$\Leftrightarrow -4x - 16y - 4z = -76$$

$$\Leftrightarrow \underline{x + 4y + z = 19}$$

· $G \in E \Leftrightarrow \underline{2x - y + 2z = 2}$

$\Big\}$ ⟹ Schnittgerade zweier Ebenen

⟹ für Schnittgerade h: $\vec{h}(t) = \vec{OM} + t\left(\begin{pmatrix}1\\4\\1\end{pmatrix} \times \begin{pmatrix}2\\-1\\2\end{pmatrix}\right) = \begin{pmatrix}2\\4\\1\end{pmatrix} + t\cdot\begin{pmatrix}8-(-1)\\2-2\\-1-8\end{pmatrix} = \begin{pmatrix}2\\4\\1\end{pmatrix} + t\cdot\begin{pmatrix}9\\0\\-9\end{pmatrix}$

$\begin{matrix}1 & 2\\4 & -1\\1 & 2\end{matrix}$

⟹ $\vec{MG} = \lambda\cdot\begin{pmatrix}9\\0\\-9\end{pmatrix}$, mit $MG = \frac{1}{2}\cdot PS$

$\downarrow$ $\quad \vec{PS} = \begin{pmatrix}1-3\\0-8\\0-2\end{pmatrix} = \begin{pmatrix}-2\\-8\\-2\end{pmatrix} \Rightarrow PS = \sqrt{72}$

$\Big\}$ ⟹ $\underline{MG} = \sqrt{\underbrace{18}_{2\cdot 9}} = \underline{3\sqrt{2}}$.

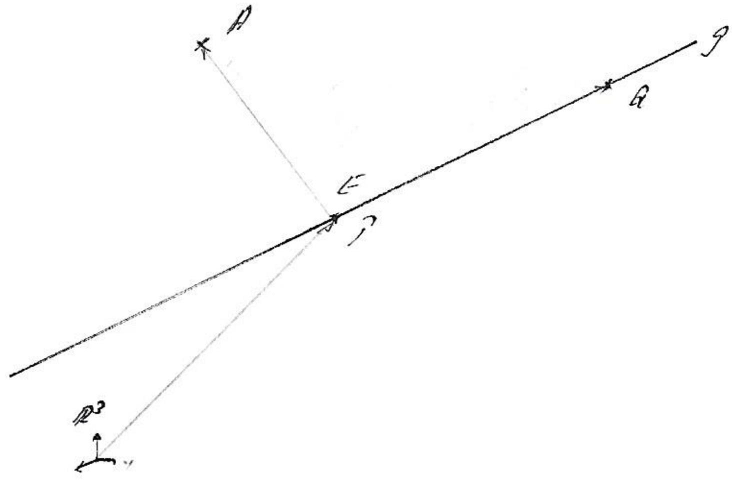
$$MG = \sqrt{81\lambda^2 + 81\lambda^2} = \lambda\cdot\sqrt{162} = \lambda\cdot 9\sqrt{2} \overset{!}{=} 3\sqrt{2}$$

$$\Leftrightarrow \underline{\lambda = 1/3}$$

⟹ $\vec{OQ} = \vec{h}(1/3) \Rightarrow \underline{Q = (5 | 4 | -2)}$

$\vec{OR} = \vec{h}(-1/3) \Rightarrow \underline{\underline{R = (-1 | 4 | 4)}}$

⑨ $A = (2/1/4)$, $P = (5/-2/16)$, $Q = (9/-9/20)$



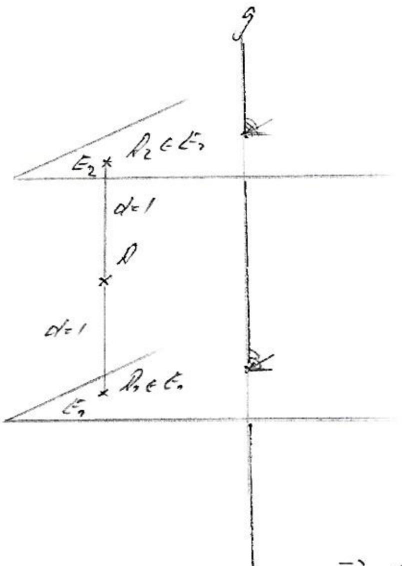
a) · Ebene E, geg. durch A und g $\Rightarrow$ $\underline{\vec{E}(r,s)} \overset{2.R}{=} \vec{OP} + r\cdot\vec{PA} + s\cdot\vec{PQ} = \underline{\begin{pmatrix}5\\-2\\16\end{pmatrix} + r\cdot\begin{pmatrix}-3\\3\\-12\end{pmatrix} + s\cdot\begin{pmatrix}4\\-7\\4\end{pmatrix}}$

$$\Rightarrow \underline{\vec{n}_E} = \begin{pmatrix}-3\\3\\-12\end{pmatrix} \times \begin{pmatrix}4\\-7\\4\end{pmatrix} = \begin{pmatrix}12-84\\-48-(-12)\\21-12\end{pmatrix} = \underline{\begin{pmatrix}-72\\-36\\9\end{pmatrix}}$$

$\begin{matrix}-3 & 4\\ 3 & -7\\ -12 & 4\end{matrix}$

· $\underline{\vec{n}_{yz} = \begin{pmatrix}1\\0\\0\end{pmatrix}}$

$$\Rightarrow \underline{\underline{\text{Schnittwinkel}}} = \cos^{-1}\left(\frac{\begin{pmatrix}8\\4\\-1\end{pmatrix}\cdot\begin{pmatrix}1\\0\\0\end{pmatrix}}{\sqrt{81\cdot 1}}\right) = \cos^{-1}\left(\frac{8}{9}\right) = \underline{\underline{27.266°}} = 0.476$$



· $E_{1,2} \perp g$ $\Rightarrow$ $\underline{\vec{n}_{E_{1,2}} = \vec{PQ}} = \begin{pmatrix}4\\-7\\4\end{pmatrix}$

· $\vec{OA}_{1,2} = \vec{OA} \pm \dfrac{\vec{n}_{E_{1,2}}}{|\vec{n}_{E_{1,2}}|} = \begin{pmatrix}2\\1\\4\end{pmatrix} \pm \frac{1}{9}\cdot\begin{pmatrix}4\\-7\\4\end{pmatrix}$

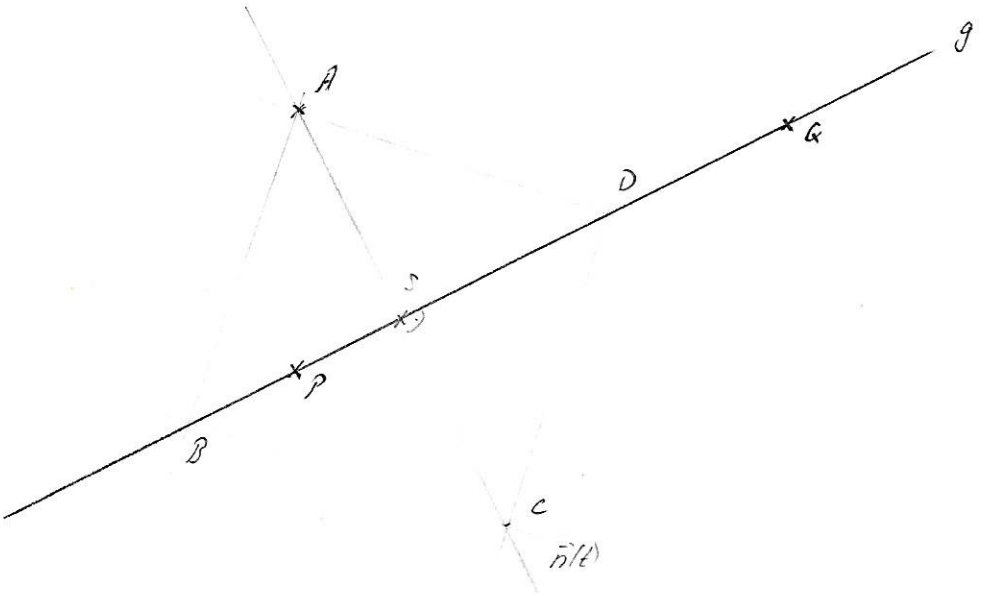
$\Rightarrow$ $\underline{A_1 = \left(\frac{22}{9} \Big/ \frac{2}{9} \Big/ \frac{40}{9}\right)}$, $\underline{A_2 = \left(\frac{14}{9} \Big/ \frac{16}{9} \Big/ \frac{32}{9}\right)}$

$= (2.444/0.222/4.444)$ $\qquad = (1.556/1.778/3.556)$

$$\left.\begin{array}{l}\Rightarrow \underline{E_1}: \quad 4x-7y+4z-d=0\\ A_1 \in E_1 \Rightarrow \underline{d=26}\end{array}\right\} \Rightarrow \underline{\underline{E_1: \; 4x-7y+4z-26=0}}$$

$$\left.\begin{array}{l}\underline{E_2}: \quad 4x-7y+4z-d=0\\ A_2 \in E_2 \Rightarrow \underline{d=8}\end{array}\right\} \Rightarrow \underline{\underline{E_2: \; 4x-7y+4z-8=0}}$$

c)



$\vec{g}(s) = \begin{pmatrix}5\\-2\\16\end{pmatrix} + s\cdot\begin{pmatrix}4\\-7\\4\end{pmatrix}$

$\vec{OP} + s\cdot\vec{PQ}$

$|\vec{PQ}| = \sqrt{16+49+16} = 9$

für $\vec{AS}$: $\cdot\ \vec{AS} = \vec{OS} - \vec{OA} = \begin{pmatrix}5+4s & -2\\-2-7s & -1\\16+4s & -4\end{pmatrix}$, $\vec{OS} \in \vec{g}$

$\cdot\ \vec{AS}\cdot\vec{PQ} \overset{!}{=} 0$

$\begin{pmatrix}3+4s\\-3-7s\\12+4s\end{pmatrix}\cdot\begin{pmatrix}4\\-7\\4\end{pmatrix} \overset{!}{=} 0$

$\Leftrightarrow 4(3+4s) + 7\cdot(3+7s) + 4\cdot(12+4s) = 0$

$\Leftrightarrow \quad \underline{s} = \frac{-81}{81} = \underline{-1}$

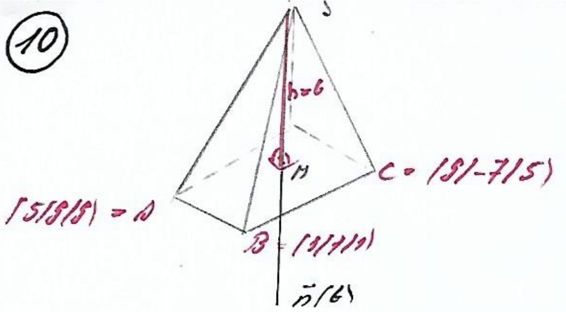
$\Rightarrow \underline{\vec{AS}} = \underline{\begin{pmatrix}-1\\4\\8\end{pmatrix}}$, $\vec{OS} = \vec{g}(-1) = \begin{pmatrix}1\\5\\12\end{pmatrix}$, $|\vec{AS}| = \sqrt{1+16+64} = 9$

$\Rightarrow \cdot\ \underline{\vec{OC}} = \vec{OA} + 2\cdot\vec{AS} = \begin{pmatrix}2+2\cdot(-1)\\1+2\cdot 4\\4+2\cdot 8\end{pmatrix} \Rightarrow \underline{\underline{C = (0/5/20)}}$

$\cdot\ \underline{\vec{OD}} = \vec{OS} + 9\cdot\frac{\vec{PQ}}{|\vec{PQ}|} = \begin{pmatrix}1+\frac{9}{9}\cdot 4\\5+\frac{9}{9}\cdot(-7)\\12+\frac{9}{9}\cdot 4\end{pmatrix} \Rightarrow \underline{\underline{D = (5/-2/16)}}$

$\cdot\ \underline{\vec{OB}} = \vec{OS} - 9\cdot\frac{\vec{PQ}}{|\vec{PQ}|} = \begin{pmatrix}1-4\\5-(-7)\\12-4\end{pmatrix} \Rightarrow \underline{\underline{B = (-3/12/8)}}$

(10)



a) **Beh.** ΔABC ist rechtwinklig - gleichschenklig

Beweis. z.z. ist $|\vec{BA}| = |\vec{BC}| \wedge \vec{BA} \cdot \vec{BC} = 0$

$$\vec{BA} = \begin{pmatrix} 5-1 \\ 9-1 \\ 9-1 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \\ 8 \end{pmatrix} \Rightarrow \underline{|\vec{BA}|} = \sqrt{16+64+64} = \underline{12}$$

$$\vec{BC} = \begin{pmatrix} 9-1 \\ -7-1 \\ 5-1 \end{pmatrix} = \begin{pmatrix} 8 \\ -8 \\ 4 \end{pmatrix} \Rightarrow \underline{|\vec{BC}|} = \sqrt{64+64+16} = \underline{12}$$

⊖ ✓

$$\vec{BA} \cdot \vec{BC} = \begin{pmatrix} 4 \\ 8 \\ 8 \end{pmatrix} \cdot \begin{pmatrix} 8 \\ -8 \\ 4 \end{pmatrix} = 4 \cdot 8 - 8 \cdot 8 + 8 \cdot 4 = 0 \quad ✓$$

b) **D & S:**

Pyramide hat regelmäßige GF $\Rightarrow$ Quadrat

$$\Rightarrow \vec{OD} = \vec{OC} + \vec{BA} = \begin{pmatrix} 9 \\ -7 \\ 5 \end{pmatrix} + \begin{pmatrix} 4 \\ 8 \\ 8 \end{pmatrix} = \begin{pmatrix} 13 \\ 1 \\ 13 \end{pmatrix}$$

$$= \vec{OA} + \vec{BC} = \begin{pmatrix} 5 \\ 9 \\ 9 \end{pmatrix} + \begin{pmatrix} 8 \\ -8 \\ 4 \end{pmatrix} = \begin{pmatrix} 13 \\ 1 \\ 13 \end{pmatrix}$$

$\Rightarrow \underline{\underline{D = (13|1|13)}}$

M = Mitte $\overline{AB}$ $\Rightarrow$ $\underline{\vec{OM}} = \frac{1}{2} \cdot (\vec{OA} + \vec{OB})$

$$= \frac{1}{2} \cdot \left(\begin{pmatrix} 5 \\ 9 \\ 9 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right) = \underline{\begin{pmatrix} 3 \\ 5 \\ 5 \end{pmatrix}}$$

$\vec{n}(t) = \vec{OM} + t \cdot \vec{n_E}$

$$\vec{n_E} = \vec{BA} \times \vec{BC} = \begin{pmatrix} 4 \\ 8 \\ 8 \end{pmatrix} \times \begin{pmatrix} 8 \\ -8 \\ 4 \end{pmatrix} = \begin{pmatrix} 8 \cdot 4 - 8 \cdot (-8) \\ 8 \cdot 8 - 4 \cdot 4 \\ 4 \cdot (-8) - 8 \cdot 8 \end{pmatrix} = \begin{pmatrix} 32+64 \\ 64-16 \\ -32-64 \end{pmatrix} = \begin{pmatrix} 96 \\ 48 \\ -96 \end{pmatrix} = 24 \cdot \begin{pmatrix} 4 \\ 2 \\ -4 \end{pmatrix}$$

$$= 48 \cdot \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$

$$\Rightarrow \vec{n}(t) = \begin{pmatrix} 3 \\ 5 \\ 5 \end{pmatrix} + t \cdot \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \qquad \Rightarrow |\vec{n}| = 6 \Leftrightarrow (3+2t)^2 + (5+t)^2 + (5-2t)^2 = 36$$

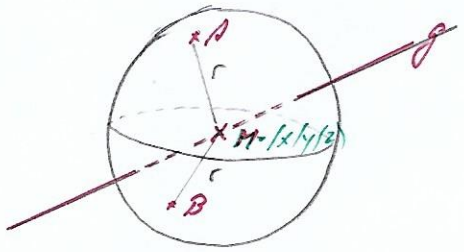
$\Leftrightarrow$

(11) Geg.: $g: \vec{r}(t) = \begin{pmatrix}8\\-2\\1\end{pmatrix} + t\cdot\begin{pmatrix}8\\-4\\1\end{pmatrix}$

$A = (-6|5|6)$, $B = (4|-5|4)$

Kugel K, mit $M \in g$, $A, B \in K$ -(Oberfläche)

a)



$A, B \in K \iff d(A,M) = d(B,M)$

$\iff (x-(-6))^2 + (y-5)^2 + (z-6)^2 = (x-4)^2 + (y-(-5))^2 + (z-4)^2$

mit $M \in g$ folgt: $M = (x|y|z) = (8+8t|-2-4t|1+t)$

$\Rightarrow (8+8t+6)^2 + (-2-4t-5)^2 + (1+t-6)^2 = (8+8t-4)^2 + (-2-4t+5)^2 + (1+t-4)^2$

$\iff (8t+14)^2 + (-4t-7)^2 + (t-5)^2 = (8t+4)^2 + (-4t+3)^2 + (t-3)^2$

$\iff 224t + 196 + 56t + 49 + (-10t) + 25 = 64t + 16 + (-24t) + 9 + (-6t) + 9$

$\iff 270t + 270 = 34t + 34$

$\iff \underline{t = -1}$

$\Rightarrow \vec{OM} = \vec{r}(-1) \Rightarrow \underline{\underline{M = (0|2|0)}}$

$\Rightarrow \underline{\underline{r}} = d(A,M) = d(B,M) = \sqrt{6^2 + (2-5)^2 + 6^2} = \underline{\underline{9}}$

3) $\measuredangle(T_1, T_2) = \measuredangle(\vec{n}_{T_1}, \vec{n}_{T_2})$

$T_1 \perp \vec{MA}$, $\vec{MA} = \begin{pmatrix}-6-0\\5-2\\6-0\end{pmatrix} = \begin{pmatrix}-6\\3\\6\end{pmatrix}$ $\Bigg\} \Rightarrow \underline{T_1: -6x + 3y + 6z - d = 0}$

$T_2 \perp \vec{MB}$, $\vec{MB} = \begin{pmatrix}4-0\\-5-2\\4-0\end{pmatrix} = \begin{pmatrix}4\\-7\\4\end{pmatrix}$ $\Bigg\} \Rightarrow \underline{T_2: 4x - 7y + 4z - d = 0}$

$\Rightarrow \measuredangle(T_1, T_2) = \cos^{-1} \dfrac{\left|\begin{pmatrix}-6\\3\\6\end{pmatrix} \cdot \begin{pmatrix}4\\-7\\4\end{pmatrix}\right|}{\sqrt{81} \cdot \sqrt{81}} = \cos^{-1}\left(\dfrac{-21}{81}\right) = 105{,}026°$

$\Rightarrow \underline{\underline{\measuredangle = 74{,}974°}}$

11 a)

mit Kugel-
gleichung

$$(\vec{x} - \vec{OM})^2 = r^2$$

$A = (-6|5|6)$

$B = (4|-5|4)$

$M \in g: \vec{g}(t) = \begin{pmatrix} 8 \\ -2 \\ 1 \end{pmatrix} + t \cdot \begin{pmatrix} 8 \\ -4 \\ 1 \end{pmatrix}$

$$\overset{\in \mathbb{R}^3}{\vec{OA} - \vec{OM}} \neq \overset{\in \mathbb{R}}{r}$$

$$\Rightarrow (\vec{OA} - \vec{OM})^2 \overset{(K)}{=} r^2 = (\vec{OB} - \vec{OM})^2$$

$$\Leftrightarrow \left(\begin{pmatrix} -6 \\ 5 \\ 6 \end{pmatrix} - \begin{pmatrix} 8+8t \\ -2-4t \\ 1+t \end{pmatrix} \right)^2 = \left(\begin{pmatrix} 4 \\ -5 \\ 4 \end{pmatrix} - \begin{pmatrix} 8+8t \\ -2-4t \\ 1+t \end{pmatrix} \right)^2$$

Wichtig, zur Lösung mit [illegible] r-Wert

$\sqrt{\ }$ nicht erlaubt! (sieht aus wie Division durch Vektor)

$$\Leftrightarrow (-14-8t)^2 + (7+4t)^2 + (5-t)^2 = (12+8t)^2 + (-3+4t)^2 + (3-t)^2$$

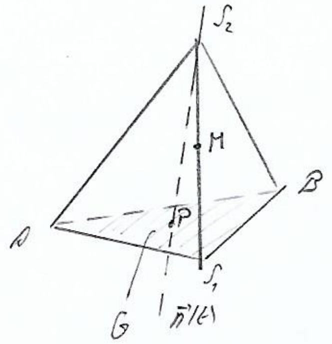
$$\Leftrightarrow 196 + 224t + 49 + 56t + 25 - 10t = 144 + 192t + 9 - 24t + 9 - 6t$$

$$\Leftrightarrow 270 + 270t = 162 + 162t$$

$$\Leftrightarrow t = -1$$

$$\Rightarrow \vec{OM} = \vec{g}(-1) = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$$

c)



• $g \cap K$: (mit Kugelgleichung)

für K: $(x-0)^2+(y-2)^2+(z-0)^2 = 9^2$

g: $\vec r(t) = \begin{pmatrix}8\\-2\\1\end{pmatrix} + t\cdot\begin{pmatrix}8\\-4\\1\end{pmatrix} \quad \left(= \begin{pmatrix}8+8t\\-2-4t\\1+t\end{pmatrix}\right)$

$\Rightarrow (8+8t)^2 + ((-2-4t)-2)^2 + (1+t)^2 = 81$

$\Leftrightarrow 64+128t+64t^2 + 16+32t+16t^2 + 1+2t+t^2 = 81$

$\Leftrightarrow 81t^2 + 162t + 81 = 81$

$\Leftrightarrow t^2 + 2t = 0 \Rightarrow t_1 = 0,\ t_2 = -2$

$\Rightarrow \vec{OS_1} = \vec r(0) = \begin{pmatrix}8\\-2\\1\end{pmatrix}, \quad \vec{OS_2} = \vec r(-2) = \begin{pmatrix}-8\\6\\-1\end{pmatrix}$

• $g \cap K$: (ohne Kugelgleichung)

Idee: $\vec{OS_i} = \vec{OM} \pm t\cdot\begin{pmatrix}8\\-4\\1\end{pmatrix}$, mit $\left|t\cdot\begin{pmatrix}8\\-4\\1\end{pmatrix}\right| = 9$

$\Leftrightarrow t = \frac{9}{\sqrt{8^2+(-4)^2+1^2}} = \frac{9}{9} = 1$

$\Rightarrow \vec{OS_1} = \vec{OM} + \begin{pmatrix}8\\-4\\1\end{pmatrix} = \begin{pmatrix}8\\-2\\1\end{pmatrix}$

$\vec{OS_2} = \vec{OM} - \begin{pmatrix}8\\-4\\1\end{pmatrix} = \begin{pmatrix}-8\\6\\-1\end{pmatrix}$

• $V_{Tetraeder} = \frac{G\cdot h}{3}$, mit $G \overset{z.B.}{=} \frac{|\vec{S_1B} \times \vec{S_1A}|}{2} = \frac{\left|\begin{pmatrix}4-8\\-5-(-2)\\4-1\end{pmatrix} \times \begin{pmatrix}-6-8\\5-(-2)\\6-1\end{pmatrix}\right|}{2}$

$= \frac12 \cdot \left|\begin{pmatrix}-4\\-3\\3\end{pmatrix} \times \begin{pmatrix}-14\\7\\5\end{pmatrix}\right| = \frac12 \cdot \left|\begin{pmatrix}-15-21\\-42-(-20)\\-28-42\end{pmatrix}\right|$

$= \frac12 \cdot \left|\begin{pmatrix}-36\\-22\\-70\end{pmatrix}\right| = \frac12 \cdot \sqrt{6680}$

für die Höhe: $h = d(S_2, E)$, mit E: aufgespannt durch D, B, S_1

für E: $\vec{S_1B} \times \vec{S_1D} = \begin{pmatrix} -36 \\ -22 \\ -70 \end{pmatrix} \Rightarrow E: 18x + 11y + 35z - d = 0$

$S_1 \in E \Rightarrow 18 \cdot 8 + 11 \cdot (-2) + 35 \cdot 1 - d = 0$
$\Leftrightarrow d = 157$

$\Rightarrow E: 18x + 11y + 35z - 157 = 0$

für n: $\vec{n}(t) = \vec{OS_2} + t \cdot \vec{n_E}$ (Normale zu E durch S_2)

$= \begin{pmatrix} -8 \\ 6 \\ -1 \end{pmatrix} + t \cdot \begin{pmatrix} 18 \\ 11 \\ 35 \end{pmatrix}$

$n \cap E$: $\Rightarrow 18 \cdot (-8 + 18t) + 11 \cdot (6 + 11t) + 35 \cdot (-1 + 35t) - 157 = 0$

$\Leftrightarrow -144 + 324t + 66 + 121t - 35 + 1225t - 157 = 0$

$\Leftrightarrow -270 + 1670t = 0$

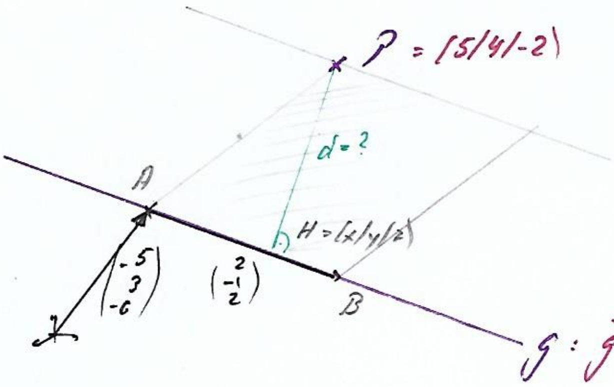
$\Rightarrow t = \frac{27}{167}$ für Höhenfußpunkt P

$\Rightarrow \vec{OP} = \vec{n}\left(\frac{27}{167}\right) \Rightarrow P = (-5{,}090 \mid 7{,}778 \mid 4{,}659)$

$\Rightarrow h = d(S_2, E) = d(S_2, P) = \sqrt{2{,}910^2 + 1{,}778^2 + 5{,}659^2} = 6{,}607$

$\Rightarrow V = \frac{G \cdot h}{3} = \underline{\underline{90}}$

(23) 'Methoden' der Abstandsbestimmung:



$$g:\ \vec{g}(t) = \underbrace{\begin{pmatrix}-5\\3\\-6\end{pmatrix}}_{\vec{OA}} + t\cdot\underbrace{\begin{pmatrix}2\\-1\\2\end{pmatrix}}_{\vec{AB}}$$

a) Mit Vektorprodukt. (=> 'unsere' Methode)

$$|\vec{AB}\times\vec{AP}| = |\vec{AB}|\cdot d$$

$$\Leftrightarrow \quad \underline{d} = \frac{|\vec{AB}\times\vec{AP}|}{|\vec{AB}|} = \frac{\left|\begin{pmatrix}2\\-1\\2\end{pmatrix}\times\begin{pmatrix}10\\1\\4\end{pmatrix}\right|}{\sqrt{9}} = \frac{\left|\begin{pmatrix}-4-2\\20-8\\2-(-10)\end{pmatrix}\right|}{3} = \frac{\sqrt{(-6)^2+12^2+12^2}}{3}$$

$$= \underline{\underline{6}}$$

b) Mit Skalarprodukt.

$$\vec{HP}\cdot\vec{AB} = \begin{pmatrix}5-x\\4-y\\-2-z\end{pmatrix}\cdot\begin{pmatrix}2\\-1\\2\end{pmatrix} \overset{!}{=} 0$$

$$H\in g \Leftrightarrow \begin{pmatrix}5-(-5+2t)\\4-(3-t)\\-2-(-6+2t)\end{pmatrix}\cdot\begin{pmatrix}2\\-1\\2\end{pmatrix} = 0$$

$$\Leftrightarrow \quad 2\cdot(10-2t) - 1\cdot(1+t) + 2\cdot(4-2t) = 0$$

$$\Leftrightarrow \quad 20 - 4t - 1 - t + 8 - 4t = 0$$

$$\Leftrightarrow \quad \underline{9t = 27} \quad \Rightarrow\ t = 3 \ \Rightarrow\ \vec{OH} = \vec{g}(3) = \begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$\Rightarrow \quad \underline{d = d(H,P)} = \sqrt{(5-1)^2+4^2+(-2)^2} = \underline{\underline{6}}\ .$$

c) Mit Pythagoras. $|\vec{AP}|^2 = |\vec{AH}|^2 + |\vec{PH}|^2$

$$\Leftrightarrow \left|\begin{pmatrix}10\\1\\4\end{pmatrix}\right|^2 = \left|\begin{pmatrix}(-5+2t)-(-5)\\(3-t)-3\\(-6+2t)-(-6)\end{pmatrix}\right|^2 + \left|\begin{pmatrix}(-5+2t)-5\\(3-t)-4\\(-6+2t)-(-2)\end{pmatrix}\right|^2 \Rightarrow t = t_0$$

$$\Rightarrow \vec{OH} = \vec{g}(t_0)$$

$H\in g \Leftrightarrow H = (-5+2t|3-t|-6+2t)$

$$\Rightarrow \underline{d = d(P,H)}$$

d) <u>Mit einer Normalenebene :</u>

$$\left.\begin{array}{l} i)\ E \perp g \text{ und } P \in E \\ ii)\ E \cap g \Rightarrow H \end{array}\right\} \Rightarrow \underline{d = d(P,H)}$$

$$\left.\begin{array}{lll} i)\ E \perp g & \Leftrightarrow & E: 2x - y + 2z - d = 0 \\ P \in E & \Leftrightarrow & E: 2\cdot 5 - 4 + 2\cdot(-2) - d = 0 \\ & \Rightarrow & d = 2 \end{array}\right\} \Rightarrow \underline{E: 2x - y + 2z - 2 = 0}$$

$$ii)\ E \cap g \Leftrightarrow 2\cdot(-5+2t) - (3-t) + 2\cdot(-6+2t) - 2 = 0$$

$$\Leftrightarrow \quad \underline{t = 3} \Rightarrow H = \ldots$$

$$\Rightarrow \underline{d = d(P,H)}$$

e) <u>Mit einer Abstandsfunktion</u>

$$d = \sqrt{(5-(-5+2t))^2 + (4-(3-t))^2 + ((-2)-(-6+2t))^2}$$

$$= \sqrt{(10-2t)^2 + (1+t)^2 + (4-2t)^2}$$

$$= \sqrt{10 - 40t + 4t^2 + 1 + 2t + t^2 + 16 - 16t + 4t^2}$$

$$= \underline{\sqrt{9t^2 - 54t + 27} = d(t) \overset{!}{=} \min}$$

$$\Rightarrow d'(t) = \tfrac{1}{2}\cdot(9t^2-54t+27)^{-1/2}\cdot(18t-54) \overset{!}{=} 0$$

$$\Leftrightarrow 18t - 54 = 0$$

$$\Leftrightarrow \underline{t = 3} \quad \left(9\cdot t^2 - 54t + 27\big|_{t=3} \neq 0\right)$$

$$\Rightarrow \vec{OH} = \vec{g}(3)$$

$$\Rightarrow \underline{d = d(H,P)}$$